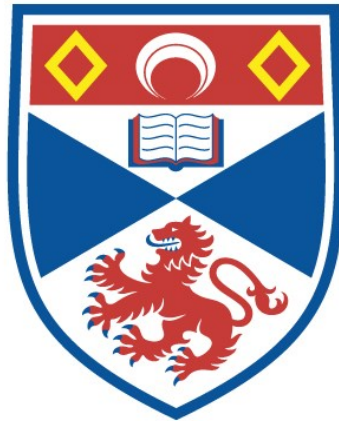


THE CONCEPT OF APPLICATION

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A Thesis Submitted for the Degree of PhD
at the
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THE CONCEPT OF APPLICATION

being a Thesis presented by

Robert H. Stoothoff

to the University of St. Andrews

in application for the degree of

Bachelor of Philosophy.



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I hereby declare that the following Thesis is based on research carried out by me, that the Thesis is my own composition, and that it has not previously been presented for a Higher Degree. The research was carried out in St. Salvator's College.

I certify that Robert H. Stoothoff has spent six terms in Research Work in St. Salvator's College, that he has fulfilled the conditions of Ordinance No. 50 (St. Andrews), and that he is qualified to submit the accompanying Thesis in application for the degree of Bachelor of Philosophy.

I matriculated in September, 1953, in Wabash College, Crawfordsville, Indiana, U.S.A., and graduated B.A. summa cum laude in June, 1957. On 8th October, 1957 I commenced research on "The Concept of Application" which is now being submitted as a Thesis for the degree of Bachelor of Philosophy. During this research I have been the holder of a Marshall Scholarship.

I.

It would perhaps be worth while to survey the various idioms which can be said to express the generic notion of using. Such an exercise in logical geography would centre on verbs like: 'use', 'employ', 'apply', 'exercise', 'practise', 'operate', 'manipulate', 'function', 'exploit', &c.; on verb phrases like: 'put to use', 'make use of', 'put into practice', &c.; and on nouns associated with these verbs and verb phrases: 'use', 'employment', 'application', 'instrument', 'method', 'utility', &c. This would be an eminently useful bit of logical geography and, I dare say, it would prove to be of philosophical significance e.g. in dealing with such puzzles as 'using words, expressions, &c.', 'following rules', 'applying mathematics', and so forth. In the course of this paper I touch upon such puzzles as these, and others like them. I do not, however, engage in logical cartography. Indeed, at times my investigation tramples somewhat rudely on niceties of English Usage: from time to time I go against usage (and, perhaps, truth as well) by forcing expressions into conceptual pigeon-holes that are not mirrored in their grammar. In other words, the general tendency of this paper is constructive rather than descriptive. My occasional use of symbolism undoubtedly emphasises the categorising aspect of my procedure.

Let us suppose a generic notion of using finds expression in locutions gathered into a set called the 'U-set'. It seems to me/

me that in going through the U-set we might come upon such diverse phrases as 'using a hammer', 'following a rule' and 'implementing a decision'. (To the objections "But we don't say 'using a decision' whereas we do say 'using a rule'", it can be answered that the OED gives a sense of 'use' in which we could say 'using a decision'.) (1) My first task is to characterise some features of the concept expressed in locutions of the U-set, assuming there to be such a concept. In view of the strong instrumental overtones of the verb 'to use', it might be preferable to think of this concept in terms of involvement in action; to think, that is to say, of the modes in which items are involved in action. Hammers, rules and decisions are in this sense involved in actions in which they are used, followed and implemented, respectively. Their mode of involvement is a function both of their nature (hammers are not like decisions) and of the nature of the action in which they are involved (using a hammer to do such-and-such is not like implementing a decision in doing so-and-so).

It might nonetheless be suggested that the notion of instrumentality provides ~~xxx~~ conceptual glue binding together diverse idioms in the U-set. There are historical grounds to support this suggestion: such verbs as 'exercise' and 'practise', for example, had a usage until the XVIIIth century permitting "He/

"He knows how to exercise (practise) the longbow" (cp. "He knows how to use (put to use, employ, &c.) the longbow"); and 'implementing a decision' or 'executing an intention' (cp. executing a plan) might seem to bear the marks of instrumentality. It is indeed part of the story, but it would be wrong to accept instrumentality as the whole of it. There is clearly a difference between e.g. using a hammer and executing a decision (cp. executing a pirouette) which cannot be ignored. Let us tentatively say that the fundamental ideas expressed in U-verbs are formulated in one or the other or both of the two formulae: 'putting... into practice' and 'putting.....to use'. We can place these ideas in opposition to one another by saying that the first formula expresses non-instrumental, and the second instrumental, involvement. This suggests a distinction among items in so far as they fit either or both of these formulae: some things can intelligibly be said to be put to use, but not put into practice (e.g. words, tools, coins, &c.); other things can intelligibly be construed in either of the formulae (e.g. rules, techniques, methods, skills, criteria, &c.); and items in a third group seem to fit (problematically) only the 'put into practice' locution (decisions, intentions and orders). To explicate this trichotomy we introduce two models: first, that provided by paradigms of (manipulative) instrumentality such as the involvement of a hammer in someone's driving nails or the use of/

in
of a scalpel/a surgical operation; and second, the involvement
of a map or itinerary in a journey taken by following it. The
instrumental model functions analogically only with respect to
non-paradigmatic cases of instrumental involvement: obviously
it cannot elucidate instances of (manipulative) instrumentality
which constitute it. But with respect to non-standard cases,
such as the involvement of 'quasi-instruments' (i.e. procedural
rules, methods, techniques, &c.) in the regulation of action, the
instrumental model plays an analogical role. The itinerary
model is meant to elucidate the non-instrumental aspect of
quasi-instrumental involvement: quasi-instruments can be regarded
in the light of either the instrumental or the itinerary model.
For example, to operate with a plan (e.g. in building a house) is
to build 'by means' of it (similarly as operating with a hammer
is performing a task by means of it), or it is to construct the
house 'in accordance' with it (analogously to following a route-map
in taking a journey). The itinerary model may also elucidate
involvement of non-instruments - items which cannot intelligibly
be said to be 'put to use'. We might, that is to say, somehow view
intentions, decisions and directives as involved in instances of
their execution, implementation or compliance similarly as
route-maps are involved in journeys. This seems a dubious analogy,
however, and I largely ignore non-instruments in what follows.
Henceforth/

Henceforth my attention is directed on instances of instrumental and quasi-instrumental involvement that are more or less paradigmatic, and I mention only such peripheral cases as may affect my investigation.

[1] Let us begin by considering in more detail the notion of instrumental involvement, i.e., the connection between an instrument and an action in which the instrument is (being) put to use. (Note that we speak of 'involvement' and of 'connection'; an underlying question is this: Are instruments (quasi-instruments) internally or externally related to actions? That is, are they involved in or linked to action?) We adopt as paradigm expression of instrumental involvement the locution 'using x (in order) to do a'; this expresses 'strong' instrumentality, and is distinguished from the locution expressive of 'weak' instrumentality 'using x in (the course of) doing a'. Thus, a carpenter's actions may be described by 'using hammer in building a house' or by '... in laying a floor'; they are also describable by 'using hammer to drive nails' or '... to straighten bent nails'. It should be observed that Ordinary Language makes no clear distinction between these two expressions of instrumental involvement; we might describe the carpenter as using a hammer in straightening nails or as using it to build a house. A general rule seems to be that a description of/

of 'weak' instrumentality is appropriate only if actions describable by expressions of 'strong' instrumentality constitute steps in the performance of the first action. That is, assuming that 'building a house' can be said to describe an action, the description 'using x in building a house' is true of NN only if there are actions A, B, C, such that 'using x to do A', 'using x to do B' and 'using x to do C' are true of NN, and by doing A, B, C, NN is building a house. Since it is usually possible in this way to reduce expressions of 'weak' to those of 'strong' instrumentality, henceforth we concern ourselves only with the latter.

One suggested analysis of the description 'using x to do a' might be 'doing a by means of x'. To say that the carpenter is using a hammer to straighten nails is just to say that he is straightening nails by means of the hammer. Or, to describe a golfer as 'using a number seven iron to make the shot' is just to describe him as making the shot by means of a number seven iron. There are, I believe, good reasons for rejecting this analysis. It may be granted that there is no logical contradiction in the conjunction of 'using the hammer to straighten nails' and 'accidentally straightened a nail', or the conjunction of 'used a number seven iron to make the shot' and 'unintentionally made the shot'./

shot's but there is extreme oddity, and (I should contend) misuse of the verb 'use'. On the other hand, there is no such oddity in the conjunction of 'straightened a nail by means of the hammer' and 'unintentionally straightened a nail', or the conjunction of 'cleaved his mother-in-law's skull with (by means of) an axe' and 'cleaved her skull accidentally'. The point is simply that using x to do a is not simply doing a by means of x, for the former implies, inter alia, an attempt to do a (I argue later that it implies more than this). This is not to deny that the 'by means of' locution occasionally carries such an implication; nor is it to deny that we might say 'use... accidentally to do so-and-so' or 'unintentionally use... to do such-and-such'. Nonetheless, for the purposes of this discussion of instrumentality I feel justified in drawing a sharp distinction between 'using x to do a' and 'doing a by means of x', and in claiming that only the first expresses the instrumental involvement of x in the performance of a. For the present we can say that 'using x to do a' entails 'trying to do a by means of x'.

This raises the question of 'task' versus 'achievement', for one criterion of so-called 'achievement verbs' is that we can always talk of trying, and failing, to do the thing in question. It might be worth asking whether verbs that can be introduced into the /

the formula 'using x to do a' satisfy another criterion, viz. whether they do not signify the occurrence of a process. Clearly they do not, for the most part, satisfy this second criterion, for it is usually legitimate to ask "How long did it take you to do a by means of x?". There are exceptions, of course, but in general actions which instrumentally involve an item are associated both with accomplishment and with performance of a task. Thus, someone describable as 'using a hammer to hit a nail' may achieve something over and above the mere nail-hitting action if he succeeds in hitting the nail with the hammer. But whether he hits it or not, he is still describable as 'using a hammer to hit a nail'. On the other hand, someone describable as 'using a hammer to drive a nail' does not achieve anything over and above his nail-driving action if he succeeds in driving it, for the former just is the latter. Action describable by 'using x to do a' has as its goal doing a, or rather the agent aims at doing a. We shall speak of this as the task in performance of which x is instrumentally involved.

Another criterion for instrumentality has been suggested by C.K. Grants:

I can say that I use something to do so and so only if I could have used something else to do the same thing. If 'use' is being used correctly it entails that there are alternatives, i.e. that there is a possibility of choice. (2)

The/

The "I could have used" in Grant's first sentence must be understood to have the force 'logically possible for me to have used'; clearly if a carpenter forgets all his saws but one, he could have used only that which he remembered, though he could have used another saw if it had been available. Furthermore, it seems to me that Grant is guilty of unauthorised linguistic legislation in claiming 'use' to be improper when there is no such possibility of choice. On the other hand, for purposes of exhibiting salient features of the concept of instrumentality, the principle that 'using x to do a' entails the (logical) possibility of 'using y to do a', 'using z to do a', &c. serves efficiently to explain our uneasiness at admitting certain cases under this concept.

For example, it provides us with a better reason for relegating 'using the mind to think' to the periphery of this concept than the reason offered by Ayer in his inaugural lecture. "The reason," he writes, "why it is not legitimate [to regard the mind as the instrument with which a person thinks or feels] is... that thinking and feeling are not the sort of things that are done with any instrument at all." (3) Again, he claims "thinking is done by persons but not with any instrument, whether the mind or any other." But he also suggests that a less misleading answer (to the/

the question 'What do you think with?') is 'I think with my brain', though he qualifies this to mean only 'that the state of our brains is a causal condition of our thinking' and adds that the brain is in no ordinary sense 'an instrument of thought'. A better reason why neither the mind nor the brain is satisfactorily characterised as 'an instrument of thought' is that there is no same-level possibility of choice among the mind (or brain) and other 'instruments'. It may be granted that there is a possibility of choice between e.g. calculating in one's head and using an adding machine, but this choice is not between alternative 'instruments' on the same (onto)logical level; whereas, for example, there is a same-level choice among different golf clubs with respect to a golf shot or among pens with respect to signing a cheque. Similarly with other bodily organs: it is improper to talk of using one's lungs to breathe or his heart to pump blood simply because there are no same-level alternatives to these organs (iron lungs and artificial hearts being analogous to an adding machine). This is not to deny that there might be such possibility of choice e.g. through evolutionary development of organs which we could use in lieu of our ordinary organs: and perhaps a situation is physiologically (and hence logically) possible in which the question "George, are you using your left or your right lung to breathe now?" has sense.

Ayer seems to accept without qualms the view that perceptual organs are instrumentally involved in 'perceptual activities', and he is tempted to affirm that we perform these activities with our brains since "there is good empirical evidence that a certain condition of the brain is causally necessary for the occurrence of any perceptual activity." (4) As to the first point, Ayer seems to make an assumption to the converse of which he traces the conception of the mind as an instrument of thought, viz. the assumption "every activity must have its special organ": that is, he argues from the premisses (1) every special organ must have its activity and (2) the perceptual organs (e.g. eyes, ears, &c) are special organs, to the conclusion (3) there are 'perceptual activities'. Aside from the fallacy which (it could be argued) is implicit in (1), however, we can reject the view that perceptual organs are instrumentally involved in any activities on the same grounds as before, i.e. just as there are no same-level alternatives to the brain, lungs or heart, in the same way there is no possibility of choice in the case of aural, visual and olfactory organs. Concerning Ayer's second suggestion, that our brains are instrumentally involved in 'perceptual activities', this is open to the same sort of objection. Ayer might argue that all he means by 'instrument' is 'causal intermediary', and that in this sense the brain and other organs are instrumentally involved in our/

our activities. This exemplifies a conceptual muddle which might be named 'the causal theory of instrumentality'; as I argue in detail later, 'using x to do a' is simply not equivalent to 'some state of x is a necessary causal condition for performance of a': the two descriptions are on a different conceptual level.

A link between instrumentality and mental activities goes back at least to Plato, and is invariably to be found in theories of the Soul and of Faculties.⁽⁵⁾ Aristotle compares the soul with the hand: the hand is τὸ ὀργάνον ὀργάνων while the mind is τὸ εἶδος εἰδῶν.⁽⁶⁾ Prichard accuses Locke of regarding the mind "as if it were a tool, of which the capacity of being used for a certain purpose should be ascertained before we use it for that purpose".⁽⁷⁾ We even find traces of it in Ryle's view that "effective possession of a piece of knowledge-that involves knowing how to use that knowledge, when required, for the solution of other theoretical or practical problems."⁽⁸⁾ Ryle seems to be claiming that knowledge-that is instrumentally involved in problem-solving. In so far as various pieces of knowledge-that (e.g. different theories) can sometimes be 'used' for solving the same problem, this case apparently satisfies the criterion of same-level possibility of choice. But to the extent that knowledge-that is transformed into knowledge-how when involved in problem-solving, it/

it is perhaps better to regard it as quasi-instrumentally than as instrumentally involved. For knowledge-how is put into practice as well as put to use.

In concluding this introductory discussion of instrumentality I am aware of having ignored several important topics. Some of these - such as the relation of causality and instrumentality - are considered later, and others - such as the work of Köhler and others on the psychology of tool-use - are omitted from consideration as not directly relevant to my development of the concept of instrumental involvement.

[2] Turning now to quasi-instrumental involvement - the involvement of rules, methods, techniques, principles, &c. in action - we must first answer this objection: Why, if quasi-instrumentally involved items can intelligibly be construed as instrumentally involved in action, should the former be distinguished from the latter? After all, the objection continues, the only difference between paradigms of instruments - hammers, pens, scalpels, &c. - and typical 'quasi-instruments' is that the former are material objects used in (for) the performance of tasks whereas the latter are immaterial 'objects' 'used' in/

in (for) the regulation of conduct; but both types of involvement - involvement in task-performance and involvement in conduct-regulation - are of the instrumental mode. It follows that the 'itinerary model' introduced above is totally unnecessary to explicate involvement of quasi-instruments in action. The burden of this objection is that e.g. to play bridge in accordance with a certain convention is to use the rules of that convention in (for) playing hands of bridge similarly as one uses a hammer in building a house. It need not be held that the rules are, themselves, instruments, but only that their involvement is adequately explained by analogy with paradigm-instances of instrumental involvement. Now I do not deny that considerable light may be thrown on the quasi-instrumental mode of involvement by comparing it with the instrumental mode; nor do I deny that it may even be feasible to regard some instances of the involvement of rules, techniques, procedures, &c. as paradigms of instrumental involvement; but I would urge nonetheless that quasi-instruments are involved analogously also as itineraries or route-maps are involved in journeys, and that since the latter mode of involvement cannot be entirely assimilated to the instrumental model it follows that its analogue, quasi-instrumental involvement, cannot be thus assimilated.

The/

The premisses of this claim seem to be plausible: that acting in accordance with a quasi-instrument is like following a map or itinerary, and that following a map or itinerary is not exhaustively (if at all) explained in terms of an instrumental analogy. On the other hand, I do not ask more of the itinerary model than that it provide a 'picture' suggestive of the difference between instrumental and quasi-instrumental involvement.

Let us ask: what is wrong with regarding a rule as instrumentally involved in an action? Grammatically, there is nothing wrong, for we can and do say e.g. that we do such-and-such with or by means of or through (the use of) a rule or method or procedure, &c. To give some examples:

- (a) Both the classical and the intuitionist calculi are designed for use in making inferences.
- (b) Of course, we cannot, when debating what criteria to use for moral grading, grade the criteria morally.
- (c) [Concept] ... a mental principle through which an individual can classify a number of objects in his stimulus world.
- (d) For instance, it is commonly stated that the rule by which a discoverer is determined, is publication...
- (e) There is no compelling a priori reason why the terminological rules which we use for describing the perceptible world must apply in all respects to images.
- (f) ...principles which... are used in arguments which start from what is experienced. ... They constitute the means of drawing inferences from what is given in sensation. (9)

Consider/

Consider (b); using criteria for grading. Urmson expresses the involvement of criteria in grading-performances by other locutions: grading is "a business done in accordance with principles";⁽¹⁰⁾ "choosing in accordance with a rule is very different in many ways from grading" though there is an analogy between "the relation of rule to choice and criteria to grade label";⁽¹¹⁾ criteria are "employed" in grading situations.⁽¹²⁾ Other expressions which Urmson uses do not explicitly mention involvement of criteria in grading performances, e.g. "criteria for grading" and "criteria for the application of grading labels"; but here, I would suggest, the 'for' can be understood as an abbreviation for 'for use in' or 'to be used for'. It seems clear that grading-criteria are more like instruments with respect to their application than are, say, moral principles with respect to action in which they are involved. At the same time, as Urmson's use of 'in accordance with' indicates, he regards criteria also as more like principles than like, say, garden hoes or scalpels. In short, he wants to say both that we grade with (by means of) grading-criteria and that we grade in accordance with them. Grading is indeed rule-guided action but it is also carried out by the use of rules as standards; and there is a same-level possibility of choice of standards just as there is a choice/

choice among garden hoes and scalpels. It will be recalled that we describe commands, decisions and intentions as 'non-instruments'; one reason for their non-instrumentality is illustrated by the fact that criteria - in common with other instruments and quasi-instruments - are involved in instances of their application, whereas directives, decisions and intentions are not involved in instances of their compliance, execution and implementation since they have no instances. Against this it may be urged that commands are unlike decisions and intentions in precisely this respect: an order can be obeyed on a number of occasions whereas a decision or an intention can be carried out or executed only once. But in so far as a command has this rule-like quality it is either the laying down or the formulation of a rule and therefore not paradigmatically a command. The specified directive - a command addressed to particular individuals explicitly prescribing an occasion of performance - is involved in the performance in question similarly as a decision in its implementation or an intention in its execution. Rule-formulating commands, on the other hand, such as doctor's orders, are involved in instances of obedience rather like rules and other quasi-instruments.

Granted, /

Granted, then, that quasi-instrumental is legitimately distinguished from instrumental involvement, we now proceed to investigate its non-instrumental features, i.e. features which make instances of quasi-instrumentality construable on the itinerary model. In this investigation it is taken as intuitively clear what it is to act in accordance with, or to follow, rules. This is a large assumption. There are rules... and there are rules. We can distinguish at least four general types of rule, ranging from recipes to statutes; ^(12a) and usage of the word "rule" - when untainted by philosophy - is so ill-delimited as almost to defy lexicographical classification. While it is perhaps to be regretted that my investigation makes extensive use of such a vague concept as 'following a rule', nonetheless I do not think that a preliminary analysis of this concept is necessary or indeed even possible. It is not necessary because I believe that intuitively - i.e. on a philosophical naïve level of thought - the concept is fairly clear; and it is not possible because such an analysis would involve sifting usages of "rule", and this would probably destroy the intuitive clarity of the concept. I hope that this does not constitute an excuse or justification for shoddy reasoning in what follows.

Though I do not sift usages of "rule", I do make a basic distinction/

distinction between two modes of rule-involvement and the types of rule peculiar to each mode. To do this I distinguish between two locutions, the R- and C-locutions, as follows:

R: 'in doing a, acting in accordance with x', or
'acting in accordance with x in doing a', or
'doing a in accordance with x';

C: 'in (by) acting in accordance with x, doing a', or
'doing a in (by) acting in accordance with x'.

These locutions are expressions of R- and C-involvement of x in doing a, and the rules, x, in each locution are accordingly designated R- and C-rules respectively. R-involvement is distinguished from C-involvement by the fact that to act in accordance with the C-rule x just is to engage in action described by 'doing a', whereas action in accordance with the R-rule x must be describable by 'doing a' apart from involvement of x. In terms of the itinerary model, an R-rule is involved in a performance similarly as one of a number of possible itineraries is involved in a journey in accordance with it, while a C-rule is involved in a performance analogously as an itinerary which is the only possible way of taking the journey. (This analogy strikes me as very unsatisfactory here, though it may nonetheless help to distinguish R- from C-involvement.) Kant's opposition of 'constitutive' and 'regulative' principles is suggestive of the distinction between C- and R-rules.⁽¹³⁾ "Constitutive" is also used in contemporary discussions/

discussions of rules, for example by Baier: "rules in this sense [sc. 'constitutive rules'] are what constitutes the nature of a certain rule-determined activity." (14) Rules of games, some legal rules, and rules definitive of rituals are commonly taken as paradigms of C-rules. In my investigation, however, the scope of 'C-rule' is considerably extended beyond these obvious cases of rule-delimited activities.

To implement this investigation I first develop, somewhat dogmatically, a certain view of human action. Let us distinguish between 'activity' and 'action' along the following lines: someone's behaviour on a given occasion (a 'behaviour-instance') is describable either as activity (in an 'activity-description') or as an action of a certain type (in an 'action-description'). For instance, my present behaviour can be described either in such terms as 'arm moving so-and-so, fingers striking keys, &c.'; or, on the other hand, in such terms as 'typing, composing a thesis, &c.'. A behaviour-instance describable by action-description D would be seen as an instance of that action by an observer competent to give and understand D as a description of any such behaviour-instance. This may seem an extraordinary claim, for it seems to put the cart (description-as) before the horse (seeing-as). That is to say, it might be objected that someone must be in a position to see X as/

as a behaviour-instance describable by D before he can describe X in terms of D; it will be argued that e.g. little children and dogs can see X as an instance of D-describable behaviour without being able to describe X by D. To answer this objection would involve a lengthy discussion of the relation between 'seeing-as' ^{uses of} and descriptive/language. Since the point is not of central importance for my argument I do not undertake such a discussion here, and would only point out that I talk above of 'seeing X as a behaviour-instance describable by D', ⁽¹⁵⁾ It is, however, of considerable importance for my argument that the notion of behaviour-qua-action is language-dependent in the way outlined above. This language-dependence connects with the notion of practice and with the intelligibility of behaviour-qua-action, as I try to show below. One further, terminological, point must be mentioned: for X, a behaviour-instance on the part of agent A, and action/activity-description D, I regard as equivalent the locutions 'describing X by D', 'describing A by D', 'describing X as D' and 'describing A as D'.

We may clarify the distinction between action and activity by the following considerations. Think of the difference between describing someone as 'making noises into an odd-shaped black object held in his hand, &c.' and as 'carrying on a telephone conversation, &c.' /

&c.' Consider the amount of explanation and instruction necessary to bring someone totally ignorant of the practice of telephone conversation to such a state of knowledge that he would see, as a matter of course, someone making noises into a black object, &c. as a person engaged in a telephone conversation, and would unhesitatingly describe him as engaged in this practice; consider, too, the fact that we do-as a matter of course, without any ado and unhesitatingly - describe and see such a behaviour-instance as an instance of this practice. The point is, that the possibility of describing behaviour-instances as actions depends on knowledge and understanding of what it is to perform such actions, and this knowledge is gained through becoming 'at home' both in engaging in practices and in dealing with other people who engage in these practices. The concept of action is intelligible only when set against a fabric of social practices. This follows from the assertion that behaviour itself is not action, but behaviour-instances can be described as actions. That is, the concept of action is dependent on the notion of description-as, but since this notion includes the possibility of misdescription, and since this possibility exists only where there are conventionally accepted criteria for what is to count as correct description, it follows that the concept of action is logically dependent on that of conventions or practices. Let us unfold/

unfold this in somewhat greater detail. For a behaviour-instance to count as an action it must be describable as an instance of a practice, or at least in terms of practices; when we speak of someone's action we do not mean simply his movements, &c. (in our sense of 'action'), for to describe his behaviour as action is to indicate that we can make sense of his movements, &c. in terms of conventionally accepted criteria by reference to which an action-description is judged to be correct or not; but these criteria 'get a grip' only in the context of agreement as to the describability of the movements, &c. Whether a particular behaviour-instance is to be described as a particular type of action depends on whether it satisfies the criteria, and that there are such criteria follows from agreement as to what is, and what is not, to count as doing such-and-such where 'doing such-and-such' names a type of action or a practice. A behaviour-instance is given action-status only if there is a practice or practices in terms of which it is intelligible, and hence describable as an action. The crucial point here is the interdependence of the concept of conventionally accepted criteria in terms of which behaviour-instances are describable (identifiable) as actions, with the concept of practices against the background of which behaviour-instances are intelligible as actions. That there are action-descriptions rests on/

on the fact that behaviour-instances are intelligible in terms of practices; or perhaps we should say that action-descriptions manifest this intelligibility. The converse may also be stated: that we find behaviour-instances intelligible follows from the possibility of describing them in terms of practices. In sum, describability is presupposed by intelligibility, and vice versa.

A word should be said about my use of the notion of 'practice'. "Practice", like "constitutive rule", has a broader usage in this paper than in other instances of its use. Rawls, for example, employs "practice" as "a sort of technical term meaning any form of activity specified by a system of rules which defines offices, roles, moves, penalties, defences and so on, and which gives the activity its structure." (16) This would approximate to my conception of 'practice' if (i) the clause about offices, roles, moves, &c. were omitted, and (ii) a clause were added connecting the 'system of rules ... which gives the activity its structure' with the conventionally accepted descriptive criteria mentioned above. A word should be said too about the word "conventional" as it occurs in 'conventionally accepted criteria': I do not mean that the criteria are agreed-upon by common consent like e.g. criteria for grading apples; the 'agreement' behind my 'conventionally accepted criteria' is, as it were, more profound than agreement by/

by common consent, in the sense that activities in which such criteria are involved serve as paradigms of agreement and cannot therefore be explained in terms of agreement. (I mention this point briefly later, in discussing Wittgenstein's treatment of 'calculating'.)

Let us now return to R- and C-involvement, to link the distinction between modes of involvement of rules in action with the view of action developed above. The C-locution, 'doing a in (by) acting in accordance with x', can be explained in this way: for anyone to behave in such a way that he would be described as 'doing a' by someone conversant in the practice of doing a, is presumptively for him to engage in this practice in (by) behaving in this way; but for him to behave in this way is for him to conduct his activities within limits correlated with conventionally accepted criteria which define the practice; that is, it is for him to act in accordance with rules expressive of these limits (which we call 'rules constitutive of a practice' or 'C-rules of a practice'); hence, we say 'in (by) acting in accordance with C-rules of the practice of doing a, one is doing a'. It is otherwise with the R-locution 'doing a in accordance with x'. An expression of R-involvement is intelligible only if the behaviour-instance in question is describable in terms of practices, since/

since to describe someone as 'doing a in accordance with x' presupposes describability by 'doing a' and this, assuming it to be an action-description, is equivalent to 'acting in accordance with C-rules of the practice of doing a'. It follows, then, that R-involvement is parasitic on C-involvement. Against this it may be objected that the behaviour-instance described by an R-locution need not be describable as an action, and only on the assumption that 'doing a' is an action-description does it follow that involvement of R-rules in behaviour-instances describable by 'doing a' is parasitic on involvement of C-rules in these instances. That is to say, behaviour describable by 'doing a in accordance with x' would not, in such a case, depend for intelligibility on understanding what it is to engage in the practice of doing a.

This objection could, I think, be answered. To answer it fully, however, would require detailed examination of various sorts of R-rules and the performances in which they are involved, showing in each case that the performance is describable by an R-locution only in so far as it is describable by an action-description (not any action description, but one correlated with practices whose C-rules stand to the R-rule in a special relation). To take just one example, consider 'assessing x's in accordance with standard S': the action-description 'assessing x's' can be/

be applied to a behaviour-instance describable by this R-locution, which is to say that S is subordinated somehow to the C-rules of the practice of assessing x's. In general, applicability of 'doing a in accordance with x' entails that either (1) 'doing a' is itself an action-description or (2) some action-description 'doing b' is applicable such that x is subordinated to C-rules of the practice of doing b (i.e. either 'doing b in accordance with x' or 'doing b in doing a' or 'doing a in doing b' is applicable). I am not prepared, at present, to attempt to justify this general principle, though it seems to me prima facie to be plausible.

[3] I want now to propose, and reject, one possible view of the nature of instrumental and quasi-instrumental involvement. This theory - let us call it the causal-interpretative theory - gives a causal account of instrumentality and locates the non-instrumental aspect of quasi-instrumental involvement in the notion of rule-interpretation.

It is tempting to think that an item instrumentally involved in an action is inso facto causally connected with it. After all, a reasonably common definition of 'instrument' is 'means by which something is caused or brought about', and it seems plausible that to use x to do a is to cause a to be done by setting x into motion/

motion or otherwise activating it. We find this view expressed by Aquinas when he says that the carpenter using a saw to create something is the principal cause of this creation while the saw is its instrumental cause; the agent is active in himself and imparts motion to the tool, thereby making it active and an instrumental cause of that of which the agent is principal cause. (17)

An instrument is really an instrument when it is an instrumental cause, i.e. when it is set in motion by some principal cause. (Aquinas argues that were there no first mover all things in motion would be instruments, but we need not pursue this interesting theological hare.) The causal theory of instrumentality can be summarised: NN is describable by 'using x to do a' if, and only if, NN's movements and (consequently) movements of x are describable by D, where describability by D is a necessary (and sufficient) causal condition of describability by 'doing a'. On this theory, the carpenter is correctly described by 'sawing a plank' only on condition that his movements and those of the saw satisfy criteria for the applicability of an activity-description 'moving (the saw) so-and-so' and of a movement-description 'the saw moving such-and-such'. Two considerations weigh against this theory. In the first place, while the action of sawing a plank is indeed circumscribed by general criteria - conventionally accepted criteria associated with C-rules of practices/

practices in terms of which behaviour describable by 'sawing a plank' is intelligible - yet these are not criteria for the applicability of activity- and movement-descriptions. To be correctly described by 'sawing a plank' it is not necessary for one to make certain movements with a saw; in so far as the causal theory implies that it is necessary, the theory is wrong.

Note that I do not deny that some movement on the part of the carpenter and the saw is necessary if 'sawing a plank' is to be correctly applied to him; I do deny, however, what the causal theory seems to imply, viz. that certain movement/activity can be specified as necessary for the correctness of 'sawing a plank'.

A second objection is this: if movement/activity descriptions are causally connected with the action-description 'doing a' as outlined above, then we should expect the former to embody a causal explanation of the latter; but it is clear that they do not. The statement "carpenter NN's movement with saw x, as described in movement/activity description D, is the cause of his sawing the plank" is difficult to construe, for NN's movement with saw x just is his sawing the plank! If the connection between NN's movement with x and his sawing the plank were causal then we might expect this statement to be an hypothesis or to express a general law about necessary concomitants of performances named by 'sawing a plank': it is not an hypothesis, however, because nothing counts as/

as testing it, nor does it express a law because it serves to explain nothing and would be of little use as a predictive device.

The connection between instrument- or agent-movements and task-performance is conceptual and not causal. Imagine a man who, in swinging an axe while cutting kindling, cleaves his wife's skull with the instrument. A number of witnesses are collected: A reports having seen him using the axe to cut kindling and accidentally catching his wife on a backswing; B reports having seen him pretending to use the axe to cut kindling but actually using it to kill his wife; C reports having seen him swinging the axe and striking his wife with it. What C comes to see or understand when he learns that the man was using the axe either to cut kindling (and accidentally struck his wife) or to kill his wife (while pretending to cut kindling), is not a causal connection between his swinging the axe and his striking his wife (in the latter case) or the lack of this connection (in the former case). To see that someone is trying to bring something about by means of an instrument is to see his behaviour in a different light, so to speak, rather than to be apprised of a special relation obtaining among agent, instrument, movement of the instrument and performance of the deed (or attempted performance)./

performance). The description 'using x to do a' implies an attempt to do a, and it also implies the quasi-instrumental involvement in this attempt of some skill or method, but it does not summarise/^{a law}or formulate a causal hypothesis to the effect that the agent's movement of x causes his performance of a.

The causal theory of instrumental involvement is complementary to an interpretative theory of quasi-instrumental involvement. I deal with one aspect of this theory later, in examining infinite regresses which it generates, but now I merely indicate how the notion of rule-interpretation fits into a causal-interpretative theory of instrumental and quasi-instrumental involvement. If, as suggested above, the involvement of a rule in action is analogous to that of a route-map in a journey, then the rule - like the map - must be interpretatively involved in action performed in accordance with it; that is, neither rule nor map can be effectively followed unless they are understood, and to understand them is to decide how they are to be followed or to interpret them. The situation resembles that of hypothetico-deductive accounts of scientific theories as meaningless (hence applicationless) formal systems until their primitive terms are interpreted through Zuordnungsdefinitionen; only when thus interpreted is the system a scientific theory which can be applied in formulating predictions, retrodictions and explanations of states of/

of affairs in terms of which its primitive terms are interpreted. Similarly, an R-rule - e.g. an instruction or a 'rule of skill' - cannot, logically, be applied by anyone who has not interpreted it in terms of the action in which it is to be involved. To interpret a map is to read it in accordance with some method of projection, and to interpret a deductive system is to understand it in terms of some Zuordnungsdefinitionen; by analogy, to interpret a rule is to grasp it by reference to a schema which intimates how it is to be followed. To follow a rule is to follow it in accordance with an interpretational schema, similarly as following a route-map involves following it in accordance with some method of projection. We are now in the jaws of a regress, since the interpretational schema does not contain in itself an indication of how it is to be quasi-instrumental involved in the action in which the rule is itself involved, and hence some schema for this schema is required. The causal-interpretative theory provides a way of escape from this regress. It agrees that the essence of quasi-instrumental involvement lies in rule-interpretation, and therefore it acknowledges the involvement of a schema in actions involving a quasi-instrument; but it denies that the schema itself is quasi-instrumentally - and hence interpretatively - involved in these performances, saying/

saying instead that the schema is instrumentally - and hence causally - involved in them. To grasp a rule is, according to this theory, to 'acquire' a schema which (causally) determines the way in which one follows the rule in subsequent actions: one 'uses' the schema in acting in accordance with the rule. The way one follows a rule could be explained in terms of ~~the~~ occurrence of certain 'brain-processes' which manifest ~~the~~ 'activation' of a rule-schema. Or, it might be held that the act of grasping the rule itself determines the way it is subsequently followed, without the mediation of a schema 'acquired' in this act.

This account does indeed stop the regress, but it breaks on the rocks of the causal theory of instrumentality. If it is implausible to hold that material instruments are causally connected with the performance of tasks, then it is even more unlikely that pieces of mental hardware such as interpretational schemata should causally effect the way in which a rule is followed. "But you have not shown this at all, for what is absurd is that schemata should be instrumentally involved in action, not that they should be causally involved: to say that schema S determines the way NN follows rule R in doing such-and-such, is not to say that NN uses S in doing such-and-such in accordance with R; your criticism of the causal theory of instrumentality therefore has no force here." But if/

if the view that schemata are somehow 'used' in determining how a rule is to be applied is given up, then schemata and rule-interpretation are unnecessary. That is to say, if a description of 'brain-processes' merely characterises an event found to coincide with behaviour described by 'following R in way I', and does not describe what happens when someone 'activates' an interpretational scheme, then we can regard R itself as instrumentally - and hence causally - involved in performances undertaken 'in accordance with R'. Hence we find an intellectualist view that the essence of quasi-instrumental involvement lies in rule-interpretation to be incompatible with the view that rule-interpretation is causally explicable. It follows from the latter view that rules themselves are instrumentally (causally) involved in action: we have already argued against both these points, viz. that quasi-instruments are only instrumentally involved in action and that instrumentality is explicable exclusively in terms of causality. We return later to consider one consequence - infinite regress - of the former view.

I should perhaps emphasise that my denial of the causal account of instrumentality is not to be construed as a repudiation of the idea that the concept of instrumentality is somehow connected with that of causality. What I object to is the view that/

that the former concept is to be explicated solely in terms of the latter. Some usages of "instrumental" are thoroughly causal, e.g. when we say "The wind was instrumental in the ship's reaching port ahead of schedule". But where human action is concerned it seems to me that the causal account is radically wrong. Instrumental involvement of items in action is a highly sophisticated phenomenon in the sense that it can adequately be treated only in terms of an essentially philosophical theory of human action. This means not only that the causal account as I have sketched it is wrong, but also that psychological theories of instrumental involvement - even those which are not behaviourist and hence basically causal - are inadequate. In this ~~latter~~ connection it is interesting to read what the behaviourist Hull writes concerning 'tool-use':

The use of tools is so natural and universal with humans that we are likely to pass this problem over without a thought, or to consider it too unimportant to merit serious consideration. A greater mistake could scarcely be made. (18)

I am in complete accord with this remark, though it seems to me that it should be addressed to philosophers as well as psychologists.

II.

In this chapter I put forward a unified characterisation of instrumental and quasi-instrumental involvement, in the sense that I study the interrelation of action-descriptions of the form 'using \underline{x} to do \underline{a} ' and those of the form '(using \underline{x} to do \underline{a}) in accordance with \underline{m} '. These are specimens of what I call 'basic U-formulae' (bufs), the former being an ' α -buf' and the latter a ' β -buf' ('doing \underline{a} in accordance with \underline{m} ' is the general form of a ' β -buf'). Thus, when I speak hereafter of instrumental or quasi-instrumental involvement, unless otherwise specified my remarks concern behaviour-instances describable by α -bufs and β -bufs. It will become clear in what follows - if it is not already evident - that the sort of instrument I most often have in mind is one that is manipulated. It will also become clear that the ' \underline{m} ' in the β -buf 'doing \underline{a} in accordance with \underline{m} ' is most often construed as a procedure (technique, method, &c.). At times I may seem to be led astray by the 'pictures' associated with procedural quasi-instrumentality and manipulative instrumentality into claiming something to be generally true for instruments and quasi-instruments that actually holds only for manipulated and methods involved in undertaking manipulative tasks. I have tried to avoid such conceptual astigmatism but it may nonetheless affect my argument.

[1] I now show that instrumental involvement entails quasi-instrumental involvement, in the sense that behaviour-instances describable by some α -buf are ipso facto describable by some β -buf. My argument proceeds on two fronts: the first is based on the intention-dependence of instrumental involvement, and involves an argument adapted from Miss Anscombe's book Intention; and the second is based on the fact that α -bufs are action-descriptions and hence linked to practices.

Someone correctly described by an α -buf must have 'non-observational knowledge' of what he is doing; that is, if he can be described by 'using x to do a ' then he can correctly answer the question "What are you doing with x ?" without 'looking to see'. Non-observational knowledge of this sort is one aspect of what can be called 'practical knowledge'. One's knowledge that some action is a causal consequence of his manipulation of an instrument is not practical knowledge: it is not a necessary condition for the correctness of 'doing a with (by means of) x ' that the agent's answer to the question "What are you doing with x ?" should manifest non-observational knowledge of what he is doing with x . For example, suppose someone to be correctly described by 'scratching the table with his pen while writing

a/

a letter', and that to the question "What are you doing with your pen?" he answers that he is (a) writing a letter and (b) scratching the table, where (a) manifests non-observational knowledge but (b) is based on his noticing that he is scratching the table with his pen while engaged in (a). Now suppose his answer (b) to be, like (a), non-observationally grounded; that is, suppose that he knows he is scratching the table with his pen in the same way that he knows himself to be writing a letter with it: can it be inferred from this that he is using the pen both to write letters and to scratch the table? In other words, is non-observational knowledge of what one is doing with x sufficient as well as necessary for the truth of a statement that one is using x to do that of which he claims to have such knowledge?

Falsity of the statement 'NN is using x to do a' would indeed seem to contradict NN's answer 'Doing a' to the question 'What are you doing with x?', where this answer ^{manifests} ~~is an expression~~ of non-observational knowledge. But does its falsity directly contradict the answer? Adapting an argument from Miss Anscombe's Intention, we say that the answer would be directly contradicted only by the performance ('What you did was a mistake, because it/

it was not in accordance with what you said."), and not by a fact that renders the description false (not: "What you said was a mistake because it was supposed to describe what you did and did not describe it."). (19) In other words, what is logically inconsistent with NN's answer "Doing a" is not falsity of the α -buf 'using x to do a', but NN's performance which this purportedly describes. Since mistakes in performance are possible it follows that practical knowledge of what one is doing with an instrument is not a sufficient condition for the truth of a corresponding β -buf: someone may be truly described by 'using x to do b' even though his answer "Doing a" to the question "What are you doing with x?" manifests non-observational knowledge of what he is doing with x. For example, one might say "Now I am using my pen to write a letter" while engaged in scratching the table with it: according to this argument, if the statement is non-observationally grounded then the mistake is not in it but in the performance (i.e. scratching the table). Miss Anscombe claims this to be analogous to obeying an order wrongly, where the fault is one of execution, not of disobedience, disregard, or ignorance of the order; though she admits the analogy to break down "where we begin to speak of knowledge." (20) I am not sure that I understand this argument completely, though

I/

I believe it supports the conclusion adduced above: practical (non-observationally based) knowledge that one is doing a with x is a necessary but not sufficient condition for truth of the corresponding α -buf 'using x to do a'.

Now, still following Miss Anscombe, we say that practical knowledge, in addition to having a 'non-observational' aspect, also is connected with "a certain sort of general capacity in a particular field," exercise of which is constituted by "nothing but the doing or supervising of the operations of which a man has practical knowledge." (21) In other words, intentional action involves the exercise of some capacity to the extent that it involves practical knowledge. In terms of instrumental involvement, we can put it this way: a necessary condition of the truth of an α -buf is the truth of some β -buf, insofar as the former presupposes 'practical knowledge'. Let us unpack this. To say that the truth of an α -buf presupposes 'practical knowledge' is to say that if 'using x to do a' is true of someone then he is able correctly to answer the question "What are you doing with x?" without need of observing his behaviour. (This is the same sort of ability as that involved in correctly answering the question "Is your middle finger straight or bent?"; it is not simply 'being able to say' such as is involved in answering the question "Where/

"Where does your leg hurt?".)(22) But practical knowledge is manifested also in one's exercise of a capacity: it has a 'knowing-how' as well as a 'knowing-that' dimension. Someone who manifests practical knowledge in its knowing-that aspect by giving a (correct) non-observationally based answer to the question "What are you doing with x", can be presumed to have the capacity to use x to do what he says he is doing with it (rather: what he knows he is doing with it). This duality of practical knowledge comes out in the sentence: "He knows what he's doing (with x)"; the question: "Do you really know what you're doing with x?" also cuts both ways, for it might mean either "Do you have the capacity to do that with x?" or "Do you know what you're doing with x?". If NN is correctly described by the α -buf 'using a saw to cut a two-by-four', then he has knowledge which is manifested both in his reply "I'm cutting this plank" to the question "What are you doing with that saw?" and in his performance. But the 'knowing-how' aspect of this knowledge is not only manifested in the performance, it is quasi-instrumentally involved in it, in so far as it consists of R-rules (of skill) in accordance with which NN is using the saw to cut the plank. Since every performance describable by an α -buf involves practical knowledge in this way, we conclude that the truth of any α -buf entails the truth of some β -buf.

We reach the same conclusion by arguing from the notion of practice, rather than (as above) from the intention-dependence of instrumental involvement. Since α -bufs are action-descriptions, it follows that any behaviour-instance describable by some α -buf can be construed as an instance of a practice; that is to say, if NN is correctly described by 'using x to do a' then it must be possible to think of him as engaged in the practice of using (something like) x to do (something like) a, whether or not there actually is such a practice. But any behaviour-instance thus describable as being conducted in accordance with the C-rules of some practice is thereby describable as being conducted in accordance with the R-rules of some method of engaging in the practice. For example, that NN is shaving implies that he is using a razor in accordance with some method of shaving with it; if NN is using a hoe to dig his garden then he must be using it to perform this task in some way, and we assume that his procedure can be expressed in terms of some method of using the hoe to dig; similarly with other cases of (manipulative) instrumental involvement. An interesting connection can be established between this demonstration of the dependence of α - on β -bufs and the demonstration based on 'practical knowledge'. To know how to use x to do a is to have learnt how to do this; but to learn how to use x to do a is, on the one hand, to learn a method (procedure) of/

of using x (to do a), and, on the other hand, to acquire skill in using x (to do a); one must, however, already possess the capacity to use x to do a (i.e. physical and mental capabilities) if one is to learn a method or to acquire skill; therefore, what one learns (acquires) is technique for (in) the exploitation of his capacity; the method learnt and skill acquired are aspects of this technique. It is important to note the distinction between possessing the capacity to use x to do a , and possessing skill in using x to do a . The knowing-how dimension of practical knowledge is basically this capacity, though we are warranted in assuming that instances of its exercise consist in the quasi-instrumental involvement of rules of skill ('technique in the exploitation of the capacity'). This is not to deny that there may be behaviour-instances describable by an α -buf where the agent has not acquired skill: but this does not entail the impossibility of formulating as rules of skill the technique by which he exploits his capacity. A person who has not learned how to use x to do a , but who has the capacity to use x to do a , surely may not succeed in his first attempt at doing a with x , but any attempt - any exercise of the capacity - can be described by some β -buf specifying a technique quasi-instrumentally involved in the action.

Ryle has argued forcefully that "there need be nothing in the performance of a solitary operation to show witnesses or even the agent himself whether the performance was an exercise of skill, prudence, taste, logical acumen or any other brand of intelligence... ways of operating are displayed not by single operations but by arrays of operations and to have a method is to operate in certain ways." (23) He goes on to distinguish "habitual actions from actions done with method," saying that the latter differ from the former in point of adaptation to problems, situations, etc. Two points are raised by Ryle's argument: first, whether single instances of instrumental involvement can be held to display some 'way of operating'; and second, whether actions describable by some α -buf can be merely habitual, and not entail the truth of some β -buf specifying a method or technique quasi-instrumentally involved in the action. I want to contend that instances of instrumental involvement both (a) display in themselves some method and (b) cannot be merely habitual; and that, in view of this, Ryle's strictures do not hold for performances described by α -bufs. Since these contentions are far from being obvious, I must say a word in defence of them. (a) As I have argued above, (manipulative) instrumental entails quasi-instrumental involvement: someone using x to do a (and not simply doing a by means/

means of x) is following a method or technique or procedure in his action. Now what are conditions under which this would be false? Presumably Ryle would argue that one might succeed in doing a, but only through luck; or that one might act habitually in using x to do a. "It is conceivable that someone should try to do a by means of x, but should succeed only by luck; consider a soldier using his rifle in target practice and accidentally scoring a bull's eye: surely this is a case of instrumental involvement without quasi-instrumental involvement." Surely it is not. The soldier is using his rifle to fire at the target, and in doing this he is trying to fire a bull's eye: some method or technique must be quasi-instrumentally involved in his using the rifle to fire at the target, though it may be granted that no method or technique need be involved in his hitting the bull's eye. Indeed, if he were to hit the bull's eye through the exercise of skill, rather than through Lady Luck, we should not say that this skill is involved in his action over and above the method or technique already involved in his using the rifle to fire at the target: the skill is this method in a perfected state. It would be odd to say that the soldier uses his rifle to fire at the target, but that he succeeds in firing at it through luck. From this I conclude that actions describable/

describable by α -bufs cannot, logically, be flukes. (b) The other case in which such an action might be held not to quasi-instrumentally involve a method or technique, is that in which it is performed habitually. It seems to me as infelicitous to talk of habitually using x to do a , as to talk of accidentally using x to do a . "But surely we say e.g. that a machine-operator on a factory assembly-line habitually or unthinkingly uses his machine to perform tasks; every time a gimcrack reaches a certain point on the conveyor-belt, he automatically presses a button, thereby using the machine to do something to the gimcrack. Or consider driving a car; we manipulate pedals and levers automatically or habitually much of the time, and in doing this we are surely using them to drive the car." Admittedly there isn't much of a method or technique involved in the machine-operator's action; but his action is only problematically described by an α -buf. Operating a machine (or an automobile) stands on the periphery of actions in which items are instrumentally involved. 'Using' a machine to perform a task for which the machine has been designed is, like 'using' one's lungs to breathe or brain to think, a degenerate case of instrumentality; as suggested above, a rough criterion for actions to be genuinely describable by α -bufs is same-level possibility of choice of instruments. As there is no choice of breathing apparatus (hence the/

the oddity of 'using one's lungs to breathe'), in the same way a machine-operator apparently has no choice of machine to do the same job: though, of course, he might have such a same-level choice. (The warning, issued above, that I take manipulative instrumental involvement as a paradigm should be repeated here. This paradigm might account for the tenacity with which I argue against 'using lungs' and 'using a complex machine': I would like to think, however, that this is not simply a case of conceptual astigmatism.) At all events, 'habitually using x to do a ' is felicitous only when 'use' has this (peripheral) sense; but where instrumental involvement is clearly in question, it is as unhappy a turn of phrase as 'accidentally using x to do a ' or 'luckily using x to do a '. We conclude, then, that Ryle's strictures do not hold for actions genuinely describable by α -bufs and that therefore instances of instrumental involvement display in themselves the quasi-instrumental involvement of some method, procedure or technique.

[2] The notion of circumstantial specification is essential not only for a unified characterisation of instrumental and quasi-instrumental involvement but also for a satisfactory account of/

of human action. After discussing this notion in both aspects I symbolise some correlative relations of circumstantially specified α - and β -bufs. To implement this latter endeavour and to aid in representing the concept of circumstantial specification, the following symbolisation is useful.

- (1) ' $\underline{x}\underline{a}$ ' stands for the α -buf 'using \underline{x} to do \underline{a} '
' $\underline{b}\underline{m}$ ' stands for the β -buf 'doing \underline{b} in accordance with \underline{m} '
- (2) ' $(\underline{x}\underline{a}/\underline{c})$ ' stands for the \underline{c} -specified α -buf 'using \underline{x} to do \underline{a} under conditions \underline{c} '
' $(\underline{b}\underline{m}/\underline{c})$ ' stands for the \underline{c} -specified β -buf 'doing \underline{b} in accordance with \underline{m} under conditions \underline{c} '
- (3) ' $(\underline{x}\underline{a}\underline{m}/\underline{c})$ ' stands for the $(\underline{m}, \underline{c})$ -specified α -buf 'using \underline{x} in accordance with \underline{m} to do \underline{a} under conditions \underline{c} ', or for the \underline{c} -specified β -buf 'using \underline{x} to do \underline{a} in accordance with \underline{m} under conditions \underline{c} '.

Interpretation of circumstantial specification - that is, how 'under conditions \underline{c} ' is to be understood - depends on one's standpoint with respect to the action. Specification of the circumstances under which an action is (being) performed can be viewed from two points of view: the spectator's and the agent's. Knowledge of the conditions under which someone is acting enables an onlooker or potential onlooker to make sense of his behaviour and to identify it as an action. From the agent's point of view, knowledge of the conditions under which he has performed or is performing enables him to correlate methods or procedures with/

with circumstantial factors, thereby to formulate rules of conduct; knowledge of the conditions under which he is to perform a task enables him to act in accordance with a method well-suited to the task under such conditions. It can be seen that the spectator-aspect of circumstantial specification has to do with the notion of practice.

Before commenting on circumstantial specification vis-à-vis the notion of practice, however, I want to distinguish between two senses of 'under conditions c'. In the first sense, it means 'under conditions of type c' (or: 'under condition-type c'), such that different performances of the same type of action can be carried out 'under conditions c'; in the second sense, it means 'under particular conditions c', such that only one performance of a given type of action can be carried out 'under conditions c'. Particular conditions are determinate both spatio-temporally and with respect to some non-spatio-temporal factors. Any action is performed under particular conditions, and whether or not a circumstantial factor has any bearing on the action, it is to be mentioned in its particular circumstantial specification. On the other hand, unless particular conditions had significant common/

common features so that performances could be specified with respect to the type of conditions under which they are carried out, knowledge of particular conditions would neither help a spectator make sense of someone's behaviour nor enable an agent to formulate rules of conduct and put them into practice. Think of the manifold of factors in a particular circumstantial specification as a fine mesh, and think of a condition-type as a pattern blocked out on the mesh: a spectator regards a behaviour-instance in the light of a number of such patterns, and as seen through one or another of them it is intelligible to him as an action; an agent, viewing his action-performed or to-be-performed through various mesh-patterns, is enabled thereby to formulate and apply rules of conduct. If no patterns recurred from one mesh to another, then circumstantial specification would be of little importance for understanding human behaviour or for effective regulation of conduct.

Whereas, of course, it is of considerable importance. Imagine a game called 'Circumstance Charades' in which only the circumstances in which someone is carrying out an action are given: the stage is set, the circumstance-mesh laid out. "R.H.S. is seated in his room with a thoughtful look on his face, there are sounds of a typewriter and of Mozart in the air, the desk by/

by his side is piled high with books and papers..." "What else could he be doing but typing his thesis?" Inference from circumstantial factors to action resembles inference from movement to action - the latter is undertaken in ordinary Charades. Just as particular conditions can display a pattern which makes behaviour intelligible as action, so movements as described in activity-descriptions may satisfy conventionally accepted criteria by which behaviour is identifiable as action of a certain type. The question "What would be the reasonable thing for someone acting under that condition-type to be doing?" is parallel to the question "What is someone making such-and-such movements likely to be doing?" It cannot be said, however, that in making those movements or in acting in those circumstances a person must be engaged in a certain type of action; nor can it be said that if he is engaged in this action then the circumstances or his movements must be like that. Wittgenstein writes: "The game, one would like to say, has not only rules but also a point." (2) Similarly, a practice is circumscribed not only by C-rules - correlated with conventionally accepted criteria for understanding and identifying behaviour in terms of the practice - but also by circumstance-patterns or condition-types, which serve to make behaviour intelligible as action by throwing light on its point. Someone/

Someone unable to identify my present action from the circumstantial specification made at the beginning of this paragraph might see immediately that I am typing when he learns of my status as a student with a thesis due shortly: this gives him a clue as to the point of my behaviour qua typing. On the other hand, clearly not all circumstantial factors have to do with the point of behaviour: the fact that there are sounds of typing in my room, for example. And, of course, many factors may be of no use in making behaviour intelligible: the fact that my room is hideously decorated or that sounds of a clock ticking, besides those of the typewriter and of Mozart, are in the air.

Turning now to circumstantial specification of (behaviour described by) α - and β -bufs, we first observe that, given both circumstances of a behaviour-instance and instrument and/or quasi-instrument involved in the behaviour, one may be able to identify the instance as action of a certain type. For example, given that someone is wielding a golf club on a well-mowed patch of lawn with a hole in the centre, it's a reasonable bet that he's putting; a person performing in accordance with The Method while on a stage is almost certainly acting. Imagine a variant of/

of the game 'Circumstance Charades' in which some circumstantial factor is picked out as having "functional or instrumental value" (Kohler), and the problem is to identify the action (being) undertaken or task (being) performed.

Yet this game, though perhaps amusing to play, would be of little practical value: a game that is of value consists in correlating an instrument and/or quasi-instrument with a given task under given conditions. The question is "How did (do) you do a under c?", the answer to which specifies an instrument and/or quasi-instrument involved, or to be involved, in doing a under c. Therefore it is misleading to talk of 'circumstantial specification', and what we have called 'circumstantially specified bufs' would better be designated 'instrumentally (quasi-instrumentally) specified task-circumstance complexes'. Instructions for a device, though mentioning conditions under which the device can be (is to be) used to perform certain tasks, correlate the device with task-and-conditions rather than the conditions with task-and-device. We must distinguish not only between particular conditions and condition-types but also between tasks and task-types, for the possibility of framing instructions and rules of conduct depends as much on the existence of sortally-related tasks as on the fact that particular conditions/

conditions share significant common features. It may be suggested that particular tasks are nothing but special circumstantial factors, since differences between tasks can often be traced to differences in the conditions under which they are to be performed. One such difference between, e.g. driving nail A and driving nail B is that performance of the first is spatially, and perhaps temporally, distinct from performance of the second; but this fact in itself may not be enough to make the first a different type of task from the second, so long as the same instrument (say, hammer x) can be used in accordance with the same method (say, method m) to drive A, as to drive B. On the other hand, some circumstantial factors, such as difference in position of nails A and B, may suffice to render driving A a different type of task from driving B; this is the case, however, only if, other circumstantial factors being more or less similar, there is no instrument-method complex (x,m) which can be (equally well) involved in driving A as in driving B. Another complication in distinguishing task-types is that while driving A and driving B may fall under the same task-type for one agent, they may be instances of quite different task-types for an agent with other capacities: NN may regard both tasks as (equally well) approachable with (x,m), whereas PP may not be able/

able to use x by m to drive A, but only to drive B. This difficulty could perhaps be surmounted by regarding agents as circumstantial factors, or even by introducing the notion of agent-type. Thus, the tasks driving A and driving B are instances of the same task-type with respect to condition-type C(NN), while only the latter is an instance of this type with respect to condition-type C(PP) and the former is an instance of some other task-type under this condition-type.

Without carrying this analysis any further we can draw the moral: the notions of task-type and condition-type are interwoven to such a degree of complexity that a separate paper would be needed to untangle them satisfactorily. It is enough, for our present purposes, to draw attention to the distinction between particular tasks and particular conditions on the one hand, and task-types and condition-types on the other. This differentiation is especially crucial in dealing with instrumental and quasi-instrumental involvement, since, as I have suggested, the very possibility of correlating instruments and/or quasi-instruments with task-circumstance complexes depends on the existence of sortally-related tasks and conditions. One further point should be mentioned before we turn to consider this correlation./

correlation. The word "complex" in the phrases 'task-circumstance complex' and 'instrument-method complex' should be understood quite literally: the notions of 'task' and of 'circumstance' are complexly interconnected, as are the notions of 'instrument' and of 'quasi-instrument'.

[3] To express the correlation of instrument and/or method with task-circumstance complex we introduce correlative formulae involving circumstantially specified α - and β -bufs. These formulae can be either normatively or non-normatively interpreted. Interpreted normatively, they express 'technical norms' which either 'prescribe' or 'permit' involvement of an instrument and/or method in the performance of a task(-type) under a condition(-type). In their non-normative interpretation they can be construed as statements to the effect either that a task is best (i.e. most efficiently) undertaken with a certain instrument and/or in accordance with a certain method, or that it is at least 'possible' to perform a task with a certain instrument and/or in accordance with a certain method. My aim here is only to schematise inter-relations among these correlative formulae.

R-formulae express rules of conduct (i.e. technical norms) which/

which 'prescriptively' correlate instruments and/or quasi-instruments (methods, techniques, &c.) with task-circumstance complexes. An instrumental R-formula, symbolised by ' $R[\underline{x}, \underline{a}/\underline{c}]$ ', expresses the rule: 'to do \underline{a} under \underline{c} , use \underline{x} ', or: ' \underline{x} is to be used to do \underline{a} under \underline{c} '; where \underline{a} is a task-type and \underline{c} a condition-type. A quasi-instrumental R-formula, symbolised by ' $R[\underline{a}^m/\underline{c}]$ ', expresses the rule: 'to do \underline{a} under \underline{c} , act in accordance with \underline{m} ', or: ' \underline{m} is to be followed in doing \underline{a} under \underline{c} '; where \underline{a} and \underline{c} are again task-type and condition-type respectively. The complex R-formula, ' $R[\underline{x}, \underline{a}^m/\underline{c}]$ ', which results from conjoining instrumental and quasi-instrumental R-formulae, seems to be ambiguous. That is, we might take the R-formula ' $R[\underline{x}, \underline{a}^m/\underline{c}]$ ' to mean (1) ' \underline{x} is to be used in accordance with \underline{m} to do \underline{a} under \underline{c} ', where only \underline{x} is prescribed; or (2) ' \underline{m} is to be followed in using \underline{x} to do \underline{a} under \underline{c} ', where only \underline{m} is prescribed; or (3) ' \underline{x} is to be used to do \underline{a} under \underline{c} , and \underline{m} is to be followed in using \underline{x} to do \underline{a} under \underline{c} ', where both \underline{x} and \underline{m} are prescribed. In what follows we are interested only in (3), in which this R-formula can be said to specify the instrument-method complex $(\underline{x}, \underline{m})$ with respect to the task-circumstance complex $(\underline{a}, \underline{c})$.

Confronted with task \underline{a} , under conditions \underline{c} in which instruments $\underline{x}_1, \underline{x}_2, \dots, \underline{x}_j$ are available, we seek the instrument-method complex such/

such that to use the instrument by the method to do a is to operate most efficiently, given conditions c . In the first place, we reject those instruments for which there is no method of using them to do a under c or if x_k is such an instrument, grounds for saying that x_k cannot be used &c. are provided by lack of any method m in accordance with which x_k can be used &c. To express this implication, we introduce correlative formulae which express the 'possibility' of an instrument and/or method with respect to a task-circumstance complex, P-formulae:

' $P[x_k a/c]$ ' for ' x_k can be used to do a under c '
 ' $P[a^m/c]$ ' for ' m can be followed in doing a under c '
 ' $P[x_k a^m/c]$ ' for ' m can be followed in using x_k to do a under c ',
 (i.e. ' m can be followed, &c., and x_k can be used, &c.').

We formulate a sufficient condition for rejecting x_k in the thesis:

$$\sim (\exists m) (P [x_k a^m/c]) \longrightarrow \sim P [x_k a/c] .$$

From this the following thesis can be deduced:

$$P [x_k a/c] \longrightarrow (\exists m) (P [x_k a^m/c])$$

This must be amended, however, for existence of a method is not the only necessary condition of validity of an instrumental P-formula (i.e. of the 'possibility' of an instrument with respect to a task-circumstance complex). Another condition is that the instrument be 'available' with respect to this complex. To say that/

that x is 'available' with respect to the complex (a, c) is to say that it is a circumstantial factor of c (or: ' $x \in c$ '). Since these necessary conditions conjointly imply the validity of the instrumental P-formula, we have the equivalence:

$$P[x \sim a/c] \longleftrightarrow x \in c \ \& \ (\exists m) (P[x \sim a^m/c])$$

In words: x can be used to do a under c if, and only if, x is available in c and some m can be followed in using x to do a under c .

We are ready now to deal with the question of specifying an optimum instrument-method complex. This amounts, in part, to consideration of the relation between R- and P-formulae, since the optimum complex, as stated in an R-formula, must be selected from the range of 'possible' complexes which are stated in P-formulae. To illustrate this, we suppose that of available x_j 's only x_3 and x_4 are 'possible': that is, there exist methods, say r and s for x_3 and t for x_4 , which can be followed in using x_3 or x_4 to do a under c . Hence we have the 'possible' complexes (x_3, r) and (x_4, t) , as stated in the P-formulae:

$$P[x_3 \sim a^r/c] \quad P[x_3 \sim a^s/c] \quad P[x_4 \sim a^t/c]$$

The question is, for which of these P-formulae is a corresponding R-formula valid? To deal with this question we must introduce the ideas of procedure and instrument-assessment.

Since/

Since instrument is definable in terms of procedure-assessment we first consider the latter. Hypothetical is to be distinguished from retrospective procedure-assessment. The former consists in judging that one method of performing a task under certain conditions would be more efficient, or otherwise suitable, than another possible method. This judgment need not precede the performance, for any action in which a method is involved can be said to manifest an hypothetical procedure-assessment. One may undertake a task in one way without being aware of the possibility of doing it in accordance with other procedures; but to undertake the performance in that way entails the judgment that one's method has certain merits, if not of efficiency then perhaps of aesthetic or some other non-procedural factor. Retrospective procedure-assessments, on the other hand, are subsequent to performances or attempted performances; though in so far as action in accordance with a certain method may reflect the adoption of a rule of conduct, retrospective assessments are displayed in performances. We represent a procedure-assessment, hypothetical or retrospective, to the effect that following m in doing a (in trying to do a) under conditions c is at least as efficient, or otherwise suitable, as following n in doing a (trying to do a) under c, by/

by the inequality-or-identity: $'[\underline{a}^m/\underline{c}] \geq [\underline{a}^n/\underline{c}]'$

Making an hypothetical assessment is like making a retroductive inference, in that for successful assessment as for fruitful retrodution a certain amount of 'insight' or perhaps 'genius' is required. The criteria by which one method is adjudged 'better' ('probably more efficient') than others, like those involved in determining one hypothesis to be 'better' ('probably more fruitful') than others, cannot be exhaustively codified into assessment- or retrodution- recipes. On the other hand, hypothetical procedure-assessments, again like retroductions, are not a-reasonable: the question "How did you know that was the best way to go about doing so-and-so under such-and-such conditions?" is not always answered by a shrug of the shoulders or a glib "I guess I'm a genius." 'Insightful behaviour', to use psychologists' jargon, does not always aptly describe performances that manifest hypothetical procedure-assessment, for often one can offer good reasons ex post facto for acting in accordanced with one method rather than another. Another danger is that procedural factors may be confused with non-procedural factors as determinants in procedure-assessments. Ryle appears to fall into this confusion when he says "to operate efficiently... is to perform one operation in a certain manner or/

or with a certain style or procedure." (25) I should have thought that 'manner' and 'style', unless manifestations of 'procedure', are to be ignored in assessing an operation for efficiency; stylistic factors are not determinants of efficient performance if they are wholly distinct from procedural factors. One of the difficulties in making retrospective procedure-assessments is to determine which stylistic factors are simply idiosyncratic, and which have procedural import. Of course some idiosyncracies are not purely stylistic, but are procedural though uncodifiable: these are flairs and geniuses, elements of Art as against Skill. In speaking here of 'optimum method' we have in mind only non-idiosyncratic procedural factors, and we assume that codifiability of methods is an overriding consideration in retrospective procedure-assessment.

Now, in terms of procedure-assessment we define instrument-assessment. An instrument, x , is to be adjudged 'better' (i.e. 'more efficient or otherwise suitable') than another instrument, y , if, and only if, there exists some method of using x to do the task in question under given circumstances such that following this method (in using x , &c.) is/adj. least as efficient, or otherwise suitable, as following any method in accordance with which y can be used/

used to do the task under the circumstances. Symbolically,

$$[x_a/c] \geq [y_a/c] \longleftrightarrow (\exists m) (n) (P[x_a^m/c] \& P[y_a^n/c] \longrightarrow [x_a^m/c] \geq [y_a^n/c]).$$

Assuming the notion of procedure- and instrument-assessment to be at least intuitively clear (which is all that is required), we next analyse R-formulae in terms of assessment and P-formulae.

First, we say that a certain method, m, is to be followed in doing b under c, if, and only if, (i) b can be done under c in accordance with m, and (ii) m is at least as efficient (or otherwise suitable) as any method which can be followed in doing b under c. That is:

$$R[b^m/c] \longleftrightarrow P[b^m/c] \& (n) (P[b^n/c] \rightarrow [b^m/c] \geq [b^n/c])$$

Similarly, we say that x is to be used to do a under c if, and only if, (i) x can be so used, and (ii) it is at least as efficient (or otherwise suitable) to use x to do a under c as it is to use any other instrument which can be so used. In symbols,

$$R[x_a/c] \longleftrightarrow P[x_a/c] \& (z) (P[z_a/c] \rightarrow [x_a/c] \geq [z_a/c])$$

We are finally in a position to combine these results and analyse the R-formula ' $R[x_a^m/c]$ ' in terms of (i) complex P-formulae of the form ' $P[z_a^n/c]$ ', and (ii) procedure-assessment with respect to task-circumstance complex (a, c) and the range of 'possible' /

'possible' instrument-method complexes. It will be recalled that we are here interested only in the interpretation of this R-formula either as a rule of conduct 'prescriptive' of both instrument and method, or, non-normatively, as a statement to the effect that of all 'possible' instrument-method complexes with respect to task-circumstance complex ($\underline{a}, \underline{c}$), ($\underline{x}, \underline{m}$) is the optimum complex in point of efficiency or other assessment-factors. On either interpretation, however, necessary and sufficient conditions for the validity of this R-formula are (i) that it be possible to use the asserted instrument, \underline{x} , to do \underline{a} under \underline{c} , (ii) that it be possible to use \underline{x} to do \underline{a} in accordance with \underline{m} under \underline{c} , and (iii) that the asserted complex be at least as efficient (or otherwise suitable) as any other complex which satisfies conditions analogous to (i) and (ii). That is:

$$R[\underline{x}, \underline{a}, \underline{m} / \underline{c}] \longleftrightarrow \underline{x} \in \underline{c} \ \& \ P[\underline{x}, \underline{a}, \underline{m} / \underline{c}] \ \& \ (\underline{z}, \underline{n}) (\underline{z} \in \underline{c} \ \& \ P[\underline{z}, \underline{a}, \underline{n} / \underline{c}] - [\underline{x}, \underline{a}, \underline{m} / \underline{c}] \geq [\underline{z}, \underline{a}, \underline{n} / \underline{c}]) .$$

If, in our example, the complex ($\underline{x}_3, \underline{s}$) is determined to be such that both $[\underline{x}_3, \underline{a}, \underline{s} / \underline{c}] \geq [\underline{x}_3, \underline{a}, \underline{r} / \underline{c}]$ and $[\underline{x}_3, \underline{a}, \underline{s} / \underline{c}] \geq [\underline{x}_4, \underline{a}, \underline{t} / \underline{c}]$, then the R-formula 'R[$\underline{x}_3, \underline{a}, \underline{s} / \underline{c}$]' can be asserted as a rule of conduct with respect to the task of doing \underline{a} under conditions \underline{c} , or as a statement to the effect that ($\underline{x}_3, \underline{s}$) is optimum instrument-method complex with respect to ($\underline{a}, \underline{c}$).

III.

I want now to return to a problem mentioned earlier: that of the possibility of infinite regress in certain accounts of quasi-instrumentality. This problem, it will be recalled, arises out of the view that following a rule entails interpreting the rule to determine how it is to be followed; since an interpretational schema is itself quasi-instrumentally involved in interpreting the rule in accordance with it, it follows that it too must be interpreted to determine how it is to be followed; hence another interpretational schema is introduced, but this also must be interpreted,... The regress, in spite of its apparent triviality, is not just of incidental significance for the notion of quasi-instrumentality: by dissecting it we may elucidate both the non-instrumental aspect of this notion and the relation between 'understanding R' and 'following R'. It is, it must be admitted, a highly artificial regress in the sense that beyond the first few steps it is difficult to make sense of its further developments: on the other hand, it is neither more nor less a mere logomachy than e.g. Lewis Carroll's puzzle about inference (to which its similarity is immediately evident). To begin with, I develop a 'double-barrelled' regress, and then I consider regress arguments in Kant, Ryle, Wittgenstein and Leibniz which bear some resemblance to either or both barrels of my regress. (26)

[1] Tom, Dick, Harry and Alf are watching a golf match. Alf has never seen golf played, and knows it only as a 'sticks-and-balls' game; Harry knows very little about golf, but he at least can tell when a player is making a shot and when he's merely addressing the ball preparatory to hitting it; Dick is more knowledgeable, for he can tell the difference between shots made in accordance with Hogan's method (H) and those made in accordance with Jones's method (J); but Tom, an eagle-eyed expert at this sort of thing, is able to distinguish among shots made by variants of these methods. That is to say, Tom's knowledge is such that, merely from the way a golfer holds his club, from the slope of his shoulders, from his follow-through, &c. he can tell whether the golfer plays in accordance with, say, H1 or H2 or H3 or J1 or J2, where 'Hn' and 'Jk' stand for method-variants of H and J respectively.

This can be seen from the agent's, as well as from the spectator's point of view. What I call the 'procedure-regress' arises on account of the theoretical possibility of indefinitely carrying out procedural specification: the spectator's specification consisting of further determination of method-variants involved in an agent's performance, and the agent's specification constituted by further variation of method and method-variants so/

so as to perform in accordance with an optimum R-rule. Suppose, for example, that NN's performance is describable by ' $\underline{x}\underline{a}\hat{H}2/\underline{c}$ ', viz. 'using club x to make shot a in accordance with method-variant H2 under conditions c'; now what is it for NN to follow H2? It is for him to follow H in a particular way, namely in accordance with variant '2. In other words, we can write the equivalence:

$$\underline{NN}\{\underline{x}\underline{a}\hat{H}2/\underline{c}\} \longleftrightarrow \underline{NN}\{\underline{x}\underline{a}\hat{H}^2/\underline{c}\},$$

where ' $\underline{NN}\{\dots/\underline{c}\}$ ' stands for 'NN performs... under c'. By supposing now that H2 and J2 have features in common which warrant specification by variant '2, and extending this supposition to procedural specification of actions in accordance with variants of methods H and J, we can say that actions in accordance with H26 and e.g. those in accordance with J16 have features in common which warrant specification by sub-variant '16. Translating this from the spectator's to the agent's point of view, we can say that anyone making a shot in accordance with H26 would be basically following method H, but that in doing this he is following variant '2, and in doing this he is following sub-variant '16. That is, we can write the equivalence:

$$\underline{NN}\{\underline{x}\underline{a}\hat{H}26/\underline{c}\} \longleftrightarrow \underline{NN}\{\underline{x}\underline{a}\hat{H}^2\ 6/\underline{c}\}$$

Imagine/

Imagine NN addressing the ball, seeking to take into account various features of the conditions, g, under which he is to make a shot: he grasps the club and sets his feet in accordance with H, but then varies his grip slightly to take account of the wind (thereby preparing to make the shot in accordance with H2), and next shifts his hips in taking account of the slope of the fairway (thereby 'setting' to follow H26 in making the shot). But there is no reason for NN to be satisfied with H26: if each subsequent variant indicates a further 'taking account of g', then he may (theoretically) continue his procedural specification ad infinitum since there is (again, theoretically) an unlimited number of features of g for him to take account of.

But there is another version of the regress. NN wonders whether he is justified in making the shot in accordance with H, rather than J: his doubts are allayed, let us say, when he remembers a rule to the effect that H is to be followed in making shots such as a under conditions such as g. But then doubts arise concerning the validity of this rule of conduct, and NN justifies his acting in accordance with it by another golfing tip which says that the rule is to be followed in conditions such as g. But following this tip must also be warranted, and its warrant justified, and its warrant's justification warranted, and... . For this, the/

the 'justification-regress', let " R_{H_1} " stand for a rule which justifies NN in following H (in using club x to make shot a under conditions o), and let " R_{H_2} " stand for a rule which warrants NN's acting on ' R_{H_1} '. The first rule is of the form: H is to be followed (or: H can be followed) in using x to do a under o; and ' R_{H_2} ' is of the form: ' R_{H_1} ' is to be followed (or: can be followed) in using x to do a under o. In making the shot in accordance with $HR_{H_1} R_{H_2}$, then, NN is said to be H-justified by ' R_{H_1} ' and ' R_{H_1} '-justified by ' R_{H_2} '. But clearly there need be no end to justification, just as there may always be further procedural specification.

The two sorts of regress can be combined into a 'double-barrelled' regress. Each choice of method must be backed by a justificatory rule; but each such rule, in turn, requires a further justificatory rule, and so on. But also, each procedural specification leads to further specification. Take NN's golf shot, for example. So far it is made in accordance with H26, where '2 and '6 are variant of H and sub-variant of H2, respectively; but it is also made in accordance with $HR_{H_1} R_{H_2}$, where ' R_{H_1} ' and ' R_{H_2} ' are rules warranting or justifying NN's following H and his justification by ' R_{H_1} ' of following H, respectively. Now justification is also needed for NN's following H2, and for his/

his following H26; but these justificatory rules - let us call them $''R_{11}''$ and $''R_{21}''$ - themselves must be warranted or backed by further rules, just as R_{H_1} had to be backed by R_{H_2} . And, of course, method-variant H26 can be further specified; but so far, what we have is this: $HR_{H_1} 2R_{H_2} R_{11} 6R_{H_3} R_{12} R_{21}$, where $''R_{H_3}$ is to $''R_{H_2}$ as $''R_{H_2}$ is to R_{H_1} , and $''R_{12}$ is to $''R_{11}$ as $''R_{H_2}$ is to R_{H_1} . It can be seen from this that, for an arbitrary primary method M, the following scheme represents the course of this 'double barrelled' regress.

$x_1, x_2, \dots, ''(n \text{ times}) x_n, \dots$: method-variants of M.

$''(n+1 \text{ times}) R_{n1}, ''(n+2 \text{ times}) R_{n2}, \dots, ''(n+k \text{ times}) R_{nk}, \dots$:

justificatory rules for x_n .

$R_{M_1}, R_{M_2}, \dots, ''(n \text{ times}) R_{M_n}, \dots$: justificatory rules for M.

$M(\infty) : M \hat{R}_{M_1} x_1 \hat{R}_{M_2} R_{11} x_2 \hat{R}_{M_3} R_{12} R_{21} x_3 \dots \hat{R}_{M_n} R_{1(n-1)} R_{2(n-2)} \dots$
 $\dots \hat{R}_{(n-1)} x_n \dots$

A similar expansion, $H(\infty)$, would represent H as (i) 'completely' procedurally specified, and (ii) 'completely' justified. Not only is every factor of circumstances g accounted for - where 'g' stands/

stands for rather more than simply NN's immediate environment - but also all the gaps are stopped, so to speak, through which doubts might enter. Presumably, if NN's performance were describable by ' $(\underline{x}, \underline{a}^H(\infty))/\underline{c}$ ' - or, for that matter, by ' $(\underline{x}, \underline{a}^J(\infty))/\underline{c}$ ' - then he would succeed in making a hole-in-one (assuming, of course, that the hole is short enough). This is all subject to the objection that it is senseless to speak of anyone - except, perhaps, God - as acting in accordance with an $M(\infty)$.

[2] The examples of regress which follow all bear resemblance to one or the other of the regresses developed above. It seems to me that the regresses throw considerable light on the nature of quasi-instrumental involvement. Indeed, it may be conjectured that this mode of involvement is essentially connected with procedure- and/or justification-regresses; I believe that Wittgenstein's preoccupation with the problem of regress in his discussion of the concept of 'following a rule' supports this conjecture.

(i) Kant draws a distinction⁽²⁷⁾ between theory and practice; the former consists of a body of practical rules which have a certain generality; and instances of practice are operations conducted in/

in accordance with rules, which manifest some purpose. An act of judgment is needed to articulate the connection between theory and instances of practice: in this act the practical man (i.e. the man who is to put theory into practice) decides whether a given situation falls under the rules of his theory. But further rules for the guidance of his power of judgment in its discriminatory exercise cannot be given indefinitely, for this would lead to infinite regress. Kant concludes from this that some men will be good in theory but unable to put their theory into practice since they lack power of judgment, and this cannot be acquired through the conning of rules. (28) Exercises of Urteilskraft are, for Kant, subsumptions under rules provided by the concepts of the understanding (Verstandesbegriffe): if these subsumptions themselves were rule-guided then a further power of judgment would be required since rule-guidance consists in subsumptive exercises of judgment; but a third-level power of judgment would be necessary if the subsumptive exercises of second-level judgment were rule-guided; and so on. Kant sets up the regress machinery in the first Kritik but does not explicitly set it into motion. (29) He hints at it also in ~~the~~ Kritik der Urteilskraft, when he discusses the difficulty of locating a priori principles of the faculty of judgment: such a principle must not serve as an "objective/

"objective rule to which it [sc. the faculty of judgment] can adapt its judgment, because for that another faculty of judgment would again be required to enable us to decide whether or not the case falls under the rule." (30) Very briefly, Kant puts a stop to the regress by claiming that for determinant (subsumptive) judgment in its transcendental mode *Urteilkraft* is guided by a priori principles of the understanding; (31) whereas in its non-transcendental exercise (e.g. subsumptions under moral principles or empirical concepts), Kant describes judgment as

... a peculiar talent which can be practised only and cannot be taught. It is the specific quality of so-called mother-wit; and its lack no school can make good. (32)

In this last statement Kant appears to be echoing Aristotle's description of *αἴσιν* as "the faculty of hitting upon the middle term instantaneously." (33)

(ii) Ryle's use of regress as a club over the heads of those who support an 'intellectualist' view of intelligent behaviour is well-known. In 'Knowing How and Knowing That' ('KH&KT') he explicitly announces his dependence on the argument "that the prevailing doctrine leads to vicious regresses", (34) and in The Concept of Mind (Concept) he appears to regard regress as "the crucial objection to the intellectualist legend." (35) In what follows/

follows I consider mainly Ryle's later regress-arguments, though I also mention some points of similarity between these and the versions in 'KH&KT'.

The first point at which regress allegedly arises has to do with the suitability or appropriateness of maxims, rules of conduct, procedures, &c. with respect to a given practical problem. The 'legend' is said to assert that "whenever an agent does anything intelligently, his act is preceded and steered by another internal act of considering a regulative proposition appropriate to his practical problem." (36) But what guarantees that the agent ~~for~~ considers an appropriate maxim? Mustn't the agent's act of consideration itself be 'preceded and steered by another internal act of considering a regulative proposition', namely a criterion of appropriateness by which to judge whether the maxim is appropriate to the task? And what about judgment with reference to the criterion of appropriateness, doesn't this require the performance of a further act of consideration? And so on. Ryle concludes:

The endlessness of this implied regress shows that the application of the criterion of appropriateness does not entail the occurrence of a process of considering this criterion. (36)

A second phase of the regress-argument concerns what Ryle calls (in 'KH&KT') the 'schizophrenic broker' or 'go-between application-process' which "has somehow to marry observance of a contemplated maxim with the enforcement of behaviour." (34) While we do not find this point explicitly set out in Concept, it seems nonetheless to lie behind Ryle's query: "[if] to act reasonably I must first perpend the reason for so acting, how am I to make a suitable application of the reason to the particular situation which my action is to meet?" (36) In other words, the mere acknowledgment of a maxim does not guarantee its correct application: maxims are to some degree general, so that suitable application in one situation may not be suitable in another even though the maxim is appropriately applied in each. The 'legend' leads to the view that between acknowledgment of an appropriate rule and appropriate application of the rule there must be an intelligent act: this act "has to unite in itself the allegedly incompatible properties of being kith to theory and kin to practice, else it could not be the applying of one in the other." (37) Summarising this aspect of the regress in Concept, Ryle writes:

A soldier does not become a shrewd general merely by endorsing the strategic principles of Clausewitz; he must also be competent to apply them. Knowing how to apply maxims cannot be reduced to, or derived from, the acceptance of those or any other maxims. (36)

The two prongs of Ryle's regress-argument resemble the two sorts of regress developed above: Ryle's 'appropriateness of maxim' phase corresponds roughly with our 'justification-regress', and his 'correctness of maxim-application' phase is similar to our 'procedure-regress'. Two points of connection between Kant and Ryle are noteworthy. In the first place, to stop the regress both introduce modes of intelligence whose exercise is practical, in the sense that neither Kant's Urteilkraft (i.e. Mutterwitz) nor Ryle's 'knowing how' are reducible to theoretical modes of intelligence. A second point is that Kant's schemata play a role similar to that played by Ryle's 'schizophrenic brokers' which marry theory and practice. A transcendental schema makes possible the application of a 'pure concept of the understanding' to 'appearances', thus serving as a 'go-between application process' which unites in itself both the intellectual (cp. theory) and the sensible (cp. practice).⁽³⁸⁾ This sort of thing is, as Ryle brilliantly shows, a natural progenitor of regress.

Ian Gallie raises the interesting question whether Ryle's regresses are logical or causal.⁽³⁹⁾ He asks whether the position which they are designed to reduce to absurdity is (a) formulated in an analytic statement "to the effect that the intelligence of any/

any practical act consists in or is reducible to the act's depending on an intelligent cognitive act", or (b) is expressed rather in a synthetic proposition "that no practical act could as a matter of fact be intelligent unless it resulted from an intelligent cognitive act." (40) Gallie argues that since (a) is "so obviously silly as to be scarcely worth refuting", Ryle must therefore be concerned to show that (b) leads to absurdity. But the regress argument then loses its force, for Gallie finds it "not... self-evident that an infinite causal series of rational acts could not, logically, be the case." (41) Furthermore, since we can understand the 'intellectualist legend' to be "maintaining that no practical act could be intelligent in the sense 'S' which applies to practical acts, without the prior occurrence of a cognitive act which was intelligent in a different and more fundamental sense 'R'", (42) it follows that the 'legend' escapes imputation of regress from the very beginning. (Gallie finds it "astonishing" that Ryle does not consider the possibility that 'intelligent' has a different sense when applied to cognitive, as against practical, acts. It is, indeed, astonishing.) I am intrigued by Gallie's suggestion that an infinite causal series of rational acts is logically unparadoxical, though I do not understand why such a series is less viciously regressive than an 'infinite/

'infinite logical series of rational acts' (i.e. a series in which the acts are links in an infinite chain of reasons à la Leibniz and Wittgenstein).

Let us apply Ryle's regress-argument to Price's claim that "intelligent action depends upon the power of recognising the situations to which one's skill or knack or expertise is applicable."⁽⁴⁾ To set the regress-machinery in motion we need only ask: Isn't recognition itself a more or less intelligent action? If one's power of recognising is involved in its exercises similarly as one's skill or knack or expertise is involved in its application, then by Price's dictum it would follow that (intelligent) recognition depends upon some further power of recognising situations to which the first power can be applied; the same might be said of this second-order power, and the regress follows. Price avoids such a reductio ad absurdum by holding that there is a basic sort of recognition ('primary recognition'), whose 'power' is innate and is exercised simply in experiencing recurrence, or in "the noticing of what is present and the memory of what is past."⁽⁴⁴⁾ On the other hand, the idea that perceptual recognition is a more or less intelligent action involving skill just as other such actions has a certain attraction.⁽⁴⁵⁾ Note, incidentally, that Price's analysis of recognition in terms of innate intelligence resembles Kant's/

Kant's reduction of Urteilskraft to unteachable Mutterwitz:
Ryle's 'knowing-how' is, of course, distinguished from both these
counters to the regress in point of its non-innateness. Though
it is not wholly relevant to the present discussion I must make
the following objection ^{against} Price's dictum. We do not apply skill to,
but rather in situations. If we can be said to apply skill to
anything (and why not?) then this would be e.g. a problem that we
try to solve by applying our skill, or a task in ~~the~~ performance of
which our skill is quasi-instrumentally involved. (Though, of
course, as I point out above in introducing 'task-circumstance
complex', a situation includes a task or problem in such a way that
the latter might be regarded as a feature of the former.)

(iii) I turn, finally, to consider Wittgenstein's treatment
of this sort of regress. The ideas of an infinite chain of
justificatory reasons and of a peculiar act of 'meaning' or
'understanding' which encapsulates an infinity of rule-applications
recur time and again in The Blue and Brown Books and Philosophical
Investigations: Wittgenstein's interlocutor seems, at times, to
be literally obsessed by these ideas, especially when he discusses
the notion of 'following a rule'.

The/

The 'chain of reasons' puzzle is succinctly stated in The Blue and Brown Books, where it is linked with remarks on reasons vis-à-vis causes and on the notion of a "mental act capable of crossing a bridge before we've got to it." (46)

Wittgenstein's argument against the idea of an infinite chain of reasons is simply that 'the chain of actual reasons has a beginning.' It is important to understand how the regress gets a grip in the first place. Wittgenstein suggests that teaching the meaning of "yellow" can be seen either as "a drill... [which] causes us to associate a yellow image, yellow things with the word 'yellow'", (47) or as "supplying a reason for doing what one [does]." (48) He rejects the first account on grounds that "the rule which has been taught and is subsequently applied interests us only so far as it is involved in the application," so that "teaching as the hypothetical history of our subsequent actions (understanding, obeying, estimating a length, etc.) drops out of our considerations." (49)

The point here is that if following a rule were causally connected with being taught the rule, then one could never know that he is following it since one's reference to the rule would always be a conjecture or hypothesis. In brief, if action-justification were merely cause-adduction then no action could ever be 'completely' justified since every 'justification' would be a causal hypothesis.

The/

The proposition that your action has such and such a cause, is a hypothesis. The hypothesis is well-founded if one has had a number of experiences which, roughly speaking, agree in showing that your action is the regular sequel of certain conditions which we then call causes of the action. In order to know the reason which you had for making a certain statement, for acting in a particular way, etc., no number of agreeing experiences is necessary, and the statement of your reason is not a hypothesis. (50)

Wittgenstein holds that there can be 'complete' justification and rational (as opposed to causal) explanation for the way one follows a rule only if 'teaching a rule' is viewed as supplying 'a rule which is itself involved in the processes of understanding, obeying, etc.; 'involved', however, meaning that the expression of this rule forms part of these processes.' (51) Wittgenstein repeats this definition of 'involve':

We shall say that the rule is involved in the understanding, obeying, etc., if, as I should like to express it, the symbol ⁽⁵²⁾ of the rule forms part of the calculation.

His usage of 'involve' is connected with a distinction between "a process in accordance with a rule" and "a process involving a rule", which gives rise to the puzzle that some behaviour-instances (e.g. writing "1, 4, 9, 16") can be described as being in accordance with a number of rules (e.g. those expressed in 'add $2n-1$ ' and 'square n '), where only one of these rules is involved in it. I digress to make a few observations about this distinction.

Wittgenstein suggests that the rules of chess are 'involved' in a game when they are expressed in a table correlating types of chess-figures with their legitimate move-patterns, which a player follows quite literally by running his finger down it to the appropriate chess-figure then across it to a move-pattern and finally moving his piece in conformity with the pattern.⁽⁵³⁾ This somewhat curious view - that "a rule, so far as it interests us, does not act at a distance" but is 'involved' in this way - reflects the methodological principle: when puzzled about 'mental activity' or about the 'objects' of this activity (e.g. wishes, images, thoughts, rules, &c.), one should substitute for the activity its expression (symbol). Thus, for the activity of thinking, substitute the expression of thoughts; for imaging substitute 'a process of looking at an object or... painting, drawing or modelling'; for a rule, substitute its tabular or graphical expression; and for rule-governed 'mental activities' like calculating, substitute 'the calculations being done on paper' along with the rule-expression as 'part of the calculation.' This seems only to be a methodological principle in Wittgenstein (a 'rule of thumb'), a procedure whereby to loosen 'mental cramps' that lead to (result from) "postulating the existence of a peculiar kind of mental act alongside of our expression."⁽⁵⁴⁾ It is not a piece of doctrine but rather, as I see it, /

it, a polemical device designed to refute the view that mental acts are essentially private. But Wittgenstein's implicit suggestion that any rule-governed behaviour can be construed as action involving its rule surely constitutes more than the mere 'loosening of a mental cramp', for it entails the doctrine that the concept of rule-governed behaviour (i.e. of 'following a rule') is intelligible only where the opposition 'correct/incorrect' gets a grip, viz. in the context of practices. Perhaps Wittgenstein's distinction between a rule involved in a performance and the performance being in accordance with the rule is meant to emphasise the public checkability that goes with the notion of practice. The distinction might, incidentally, be compared with Kant's opposition of action from duty to action in conformity with duty: if NN's action involves rule R (is done from duty D) then it is in accordance with R (in conformity with D), whereas it does not follow from the fact that NN's action is in accordance with R (in conformity with D) that it involves R (that it is done from D).⁽⁵⁵⁾ One further point: my usage of 'involve' and 'involvement' has nothing to do with Wittgenstein's.

To return from this digression to the subject of regress, I want to re-consider the notion of rule-interpretation in the light of some passages/

passages in Investigations. The interlocutor at one point complains:

"But how can a rule shew me what I have to do at this point? Whatever I do is, on some interpretation, in accord with the rule." (56)

Wittgenstein seeks to dissolve the comundrum by pointing out "if everything can be made to accord with the rule, then it can also be made out to conflict with it. And so there can be neither accord nor conflict here." (57) The puzzle results from our inclination to associate an act of rule-interpretation with every instance of rule-involvement; on the contrary, Wittgenstein argues, if there were not "a way of grasping a rule which is not an interpretation, but which is exhibited in what we call 'obeying the rule' and 'going against it' in actual cases," (58) then not only would the paradoxical consequence (i.e. 'neither accord nor conflict') follow, but also the concept of following and violating rules would be rendered pointless. What gives this concept point is the opposition 'correct/incorrect', and this is grounded in the connection of rule-governed behaviour with action-descriptions and hence with practices: it is not based on the idea of rule-interpretation. A sign-post is 'the expression of a rule', and "a person goes by a sign-post only in so far as there exists a regular use of sign-posts, a custom." (59)

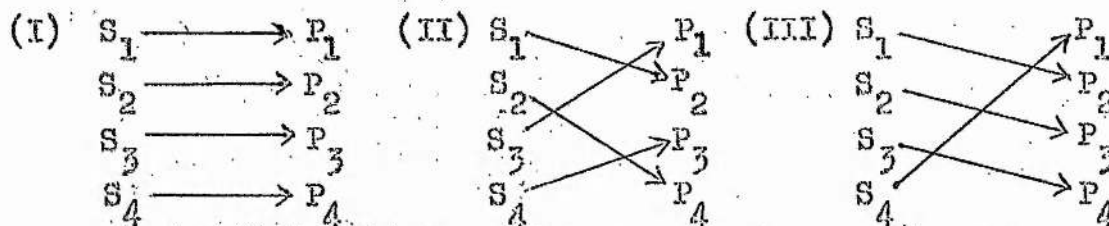
The view that rule-involvement entails rule-interpretation leads, /

leads, as we have already seen, to regress. It is a form of Scepticism: the question of rule-interpretation arises only when there is doubt as to how a rule is to be followed, and if every behaviour-instance describable as action in accordance with a rule were accompanied by an act of interpreting the rule then resolution of doubt would be essential to the concept 'following a rule'. Wittgenstein deals with some ramifications of this view in these passages of Investigations.

- (a) I said that the application of a word is not everywhere bounded by rules. But what does a game look like that is everywhere bounded by rules? whose rules never let a doubt creep in, but stop all the cracks where it might? - can't we imagine a rule determining the application of a rule, and a doubt which it removes - and so on?
- (b) A rule stands there like a sign-post. - Does the sign-post leave no doubt open about the way I have to go?
- (c) It may very easily look as if every doubt merely revealed an existing gap in the foundations; so that secure understanding is only possible if we first doubt everything that can be doubted, and then remove all these doubts. The sign-post is in order - if, under normal circumstances, it fulfils its purpose. (60)

An example will be helpful. Think of a table consisting of two columns each with n signs: let us call this an n-element table. Various ways to read such a table can be set out in n-element schemata here/

here are three typical 4-element schemata:



The table, let us say, expresses a rule R, and its schemata are interpretative rules one of which is involved in any action in accordance with R. (Imagine, for example, that the table is a railway time-table: then the question "How do you read this table?" is answered by "Read it in accordance with (i.e. interpret it in accordance with) schema (I).") At this point Wittgenstein would ask:

Can we not imagine further rules to explain these?
 And, on the other hand, was that first table incomplete without the schema of arrows? And are other tables incomplete without their schemata? (61)

That is, even if (I), (II) or (III) were supplied with the table, there might still be doubt as to how to follow the schema; but this doubt arises only if it is thought that the schema must itself be interpreted in so far as it is followed. While such 'second-order' doubts might indeed arise, nonetheless if there were no doubt-free instances of acting in accordance with R then the table would no longer express R: it plays a rule-expressive role only because there are actions in which it is followed in one way or another as a matter of

of course, without doubts requiring interposition of an interpretative rule for their resolution. What would it be like, though, to 'first doubt everything that can be doubted' about the 4-element table, 'and then to remove all these doubts' so as to 'secure understanding'? Given the command "Correlate S_k ", once having determined (through resolution of four doubts) that S_k is to be correlated with P_m , then six doubts must be resolved in determining which of six schemata are to be followed in carrying out the order. Hence, to 'follow R' in obeying the command initially involves twenty-four doubt-resolutions. But a question may remain about the supposedly doubt-free schema finally determined: can this not be variously interpreted? This question is resolved by introducing secondary schemata in accordance with which schemata such as (I), (II) and (III) can be interpreted. In fact, of course, secondary schemata must already have been introduced, for resolution of doubts about the interpretation of primary schemata entails consideration of alternative schemata-i interpretations; similarly, each secondary schema must be associated with tertiary schemata, and so on. Here is a selection of n -ary schemata and $n + 1$ -ary interpretative rules for a 4-element table:

<u>n + 1-ary in- interpretative rules</u>	<u>n-ary schemata</u>			
"Up-one"				
"Down-two"				
"1-3 switch"				

Interpretation of an n-ary schema in accordance with ^{an} n+1-ary rule results in the n+1-ary schema as shown. Incidentally, a double-barrelled regress arises in the case of a table with infinitely many correlatable items, since for this table an infinity of doubts must be resolved at the outset (i.e. at the primary level) as against just twenty-four for the 4-element table.

Wittgenstein is content, however, with a single-barrelled regress. He writes, in The Blue Book:

Now there is the idea that if an order is understood and obeyed there must be a reason for our obeying it as we do; and in fact, a chain of reasons reaching back to infinity. This is as if one said: "Wherever you are, you must have got there from somewhere else, and to that previous place from another place; and so on ad infinitum." (If, on the other hand, you had said, "wherever you are, you could have got there from another place ten yards away; and to that other place from a third, ten yards further away, and so on ad infinitum", if you had said this you would have stressed the infinite possibility of making a step. Thus the idea of an infinite chain of reasons arises out of a confusion similar to this: that a line of a certain/

certain length consists of an infinite number of parts because it is indefinitely indivisible; i.e. because there is no end to the possibility of dividing it.) (62)

He goes on then to claim that once the idea of an infinite chain of reasons is eliminated - that is, once it is acknowledged that the chain comes to an end - then the idea of obeying an order (following a rule) blindly, having no reason for the way one obeys (follows) it, is no longer found 'revolting'. What does Wittgenstein mean by 'a reason for doing what one does'? He suggests that "giving a reason for something one did or said means showing a way which leads to this action"; (63) and again, "giving a reason is like giving a calculation by which you have arrived at a certain result." (64) He distinguishes, however, between "telling the way which one has gone himself" and "describing a way which leads there and is in accordance with certain accepted rules" (a 'justification post hoc'). (65) This distinction seems to parallel the distinction between action involving a rule and action in accordance with a rule: to 'tell the way one has gone' is to give a rule involved in action, whereas in giving a 'justification post hoc' one may state only a rule with which one's action is in accordance. It is not clear which of these types of justificatory rule (reason) is embodied in the 'chain of reasons reaching back to infinity'./

infinity'. Nor, for that matter, is it yet clear how the regress gets started in the first place.

To throw some light on the latter problem we must attend to a point mentioned earlier:

If... you realise that the chain of actual reasons has a beginning, you will no longer be revolted by the idea of a case in which there is no reason for the way you obey the order. (66)

That is to say, once the idea of an infinite chain of reasons is destroyed then the "general disease of thinking which always looks for (and finds) what would be called a mental state from which all our acts spring as from a reservoir" (67) is cured. One symptom of this disease is the view that 'some justifying mental act' stands behind 'the way you obey the order' in each instance of your obedience (cp. 'the way you follow the rule' in each instance of your applying it). This mental act encapsulates an infinite chain of reasons, one for each execution of the order (application of the rule) in a particular way. This can be illustrated in terms of an argument in The Brown Book. Wittgenstein invites us to suppose that a pupil, in executing the order "Add 1", begins doing what we should call 'adding 2' after passing 100 and begins adding 3 after 200, &c. Wittgenstein says:

We/

We might in such a case say that this person naturally understands (interprets) the rule (and examples) we have given as we should understand the rule (and examples) telling us: "Add 1 up to 100, then 2 up to 200, etc." (68)

Next the question is raised: What is it for someone to follow the order (rule) correctly? The interlocutor's suggestion "the correct step at every point is that which is in accordance with the rule as it is meant, intended" (so. by the person who issues the order "Add 1") provokes Wittgenstein to ask "But how did you do all these acts of meaning (I suppose an infinite number of them) when you gave him the rule?" (69) Alternatively, how could all these acts of meaning follow from, or be embodied in, the single act of meaning which allegedly accompanies giving the rule? The interlocutor is accused of mystery-mongering:

Your idea really is that somehow in the mysterious act of meaning the rule you made the transitions without really making them. You crossed all the bridges before you were there. (70)

Again, Wittgenstein attacks the interlocutor's suggestion:

The expression "The rule meant him to follow up 100 by 101" makes it appear that this rule, as it was meant, foreshadowed all the transitions which were to be made according to it. But the assumption of a shadow of a transition does not get us any further, because it does not bridge the gulf between it and the real transition. If the mere words of a rule could not anticipate a future transition, no word could any mental act accompanying these words. (71)

This/

This mental act is correlated with "an act of insight, intuition, which makes us use the rule as we do at the particular point of the series";⁽⁷²⁾ this is the act of understanding or interpreting or grasping the rule (order), the 'justifying mental act' standing behind action in accordance with it. Wittgenstein makes the point here, and also in Investigations, that "it would be less confusing to call it an act of decision"; on the other hand, "nothing like an act of decision must take place, but possibly just an act of writing or speaking."⁽⁷³⁾ (Compare Ryle's 'go-between application-process'.) In concluding his argument Wittgenstein asserts: "We need have no reason to follow the rule as we do. The chain of reasons has an end."⁽⁷⁴⁾

If one denies the necessity of an infinite chain of reasons then one must admit the possibility of a case without even a finite chain of reasons; for if the chain of reasons has an end then one may have no reason for the way he follows a rule or obeys an order. But to have no reason is to follow the rule as a matter of course in one way rather than another. Because it is a matter of course to continue the series '100, 101, 102, ...' rather than '100, 102, 104, ...', the order (rule) "Add 1" may "seem to produce all its consequences in advance!"^(74a) 'Doing something as a matter of course' plays a prominent role in Wittgenstein's phenomenological analysis/

analysis of the concept 'following a rule': it is, in a way, the basis of what he calls "this curious superstition... that the mental act is capable of crossing a bridge before we've got to it,"⁽⁷⁵⁾ for we fall into this delusion in seeking to account for our 'matter of course' attitudes and activities but failing to see the bedrock of conventions and practices - the "forms of life" - that underlie them.⁽⁷⁶⁾ "One does not feel that he has always got to wait upon the nod (whisper) of the rule."⁽⁷⁷⁾ To account for this phenomenon we suppose that 'all the steps are really already taken' and assume that "the rule, once stamped with a particular meaning, traces the lines along which it is to be followed through the whole of space."⁽⁷⁸⁾ In doing this, however, we are looking precisely in the wrong direction, for it is in 'agreement in forms of life' rather than in mental acts incapsulating 'the unlimited application of a rule' that the concept 'following a rule' finds its Heimat.

"How am I to obey a rule?" - if this is not a question about causes, then it is about the justification for my following the rule in the way I do.

If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what I do."⁽⁷⁹⁾

The chain of reasons has an end: it therefore need have no beginning. Except in non-normal cases (i.e. cases where there is genuine doubt) there is no question of justification, and hence no/

no question of a 'justifying mental act': justifications are, in Wittgenstein's sense, 'post hoc' in that they refer to 'certain accepted rules' and not to 'the way which one has gone himself'.

(iv) In conclusion I want to raise the question: Is a justification-regress necessarily vicious? Ian Callie, it will be recalled, found nothing paradoxical about the notion of an 'infinite causal series of rational acts': here I want to enquire into the regressum ad infinitum associated with an infinite logical series, i.e. an infinite chain of reasons. C.K. Grant assumes that such a series of justificatory rules (or, perhaps, procedural rules) is viciously regressive: "a rule does not require for its application a further rule; to suppose that it did would be to generate a vicious infinite regress."⁽⁸⁰⁾ But suppose that the infinite rule/reason-series $R_1 \wedge R_2 \wedge R_3 \dots \wedge R_n \dots$ where R_1 somehow 'requires' R_{1+1} for its application, is analogous to a convergent infinite series such as $1 + \frac{1}{2 \cdot 1} + \frac{1}{3 \cdot 2 \cdot 1} + \dots + \frac{1}{n!} + \dots$. Then we might say that the regress of rules/reasons is not vicious, since action in accordance with the rule/reason-series would quasi-instrumentally involve the Super-rule, R , on which the series converges. It has been suggested by Rescher⁽⁸¹⁾ that Leibniz's doctrine of contingency depends on precisely this analogy; /

analogy; and though it may legitimately be objected that to make the notion of a convergent rule-series intelligible for God is not to make it intelligible for man, yet I think that Rescher's interpretation of Leibniz is nonetheless interesting as a suggestion of how a justification-regress might not constitute a reductio ad absurdum. For this reason, and for the further reason that no discussion of the 'chain of reasons' would be complete without mentioning Leibniz, I want to sketch Rescher's argument.

He starts from the premiss that Leibniz distinguishes between the moral and the metaphysical perfection of God. The former is expressed in the Principle of Perfection, that God selects the best of all possible worlds or that His acts (the sphere of his activity being the world) are the best possible. God's metaphysical perfection, His possession of the maximum amount of essence, follows from God's necessary existence which is proved, Rescher argues, not by the Anselmian-Cartesian Ontological Argument (which premises God's perfection) but by (what Rescher calls) the 'Modal Argument'.⁽⁸²⁾ God's moral perfection is contingent (since His choice of 'the best of all possibles' is not necessitated) while His metaphysical perfection is necessary (being/

(being a consequence of His necessary existence). But God's moral perfection must then have a sufficient reason, and this in turn another, and so on to infinity: the truth of the proposition asserting God's moral perfection is established by an infinite analysis of the notion of God, for this notion comprises an infinite series of reasons for His moral perfection similarly as the notion of Caesar comprises an infinite series of reasons for his crossing the Rubicon rather than stopping at it. Next Rescher claims that this series of sufficient reasons converges on God's metaphysical perfection; his support for this all-important step is that "the (infinite) analysis of the contingent must ultimately lead to the necessary, i.e. to God qua metaphysically perfect."⁽⁸³⁾ Thus, God's action in accordance with the Principle of Perfection is ipso facto warranted by an infinite series of sufficient reasons; but since this series converges on His metaphysical perfection, it follows that His acts in accordance with the Principle are logically intelligible while not logically necessitated. In this way, incidentally, Rescher claims that Leibniz can maintain, without contradiction, both the contingency of God's goodness and the necessity of His existence. Rescher attaches considerable importance to the place of convergence in Leibniz's philosophy, saying that Leibniz "had mastered

a lesson which philosophy was slow to learn - that infinite processes are not ipso facto vicious, since convergence is possible." (84). It is not altogether clear to me, however, precisely how the convergence of this infinite chain of reasons is analogous to mathematical convergence, unless it be that each reason is correlated with an amount of perfection so that the 'sum' of reasons converges on the maximum amount of perfection as found in God's metaphysical perfection. Nor am I clear how to understand this analogy in terms of acts other than those of God. (84a)

IV.

The concept of application is associated with a mode of involvement that cuts across instrumental and quasi-instrumental involvement. This mode we call 'applicative involvement', and we talk of items as 'applicatively involved' in actions and activities. In addition to an element of instrumentality, or quasi-instrumentality, this mode of involvement embodies also an element of 'referentiality'. Both aspects are evident in the grammar of the verb 'to apply... to...'. To give some idea of the scope of applicative involvement - and hence of the concept of application - I append a tripartite list of sentences exemplifying (I) application of words, expressions, predicates, &c. to objects, persons, events, actions, &c.; (II) application of concepts, criteria, standards and legal rules to objects, persons, actions, cases, &c.; and (III) application of theoretical, mathematical and logical systems, and their components (i.e. hypotheses, laws, rules, &c.) to objects, events, phenomena, calculations, &c.

- (I) (a) The peculiarity of dispositional predicates is that they seem to be applied to things in virtue of possible rather than actual occurrences -
- (b) And the only relation of a term to a thing is that of applicable or not applicable.
- (c) Such is even the word whiteness, in respect of the different shades of whiteness to which it is applied in common;
- (d) The word 'table'... denotes the objects to which it is applicable...

(II)/

- (II) (a) We must be able to show how pure concepts of the understanding can be applied to appearances.
- (b) Like verdicts, both moral and non-moral appraisals usually consist in the application of accepted rules, principles, and criteria to a particular case.
- (c) Finally, and generally, a particular evaluation is the application to a particular case of a standard of preference.
- (d) If he [sc. a policeman] did apply the no parking rule to the motorist, he would be applying it where it does not apply, because this is one of the recognised exceptions which are part of the rule.
- (e) The latter [sc. 'law-applying organs'] by their acts create individual norms, thereby applying the general norms to concrete cases.
-
- (III) (a) The lowest-level hypothesis IIIa. is tested by applying it to a particular case.
- (b) Classical physics is so general that no conceivable physical event falls outside the domain of its applicability.
- (c) Every logical formula, being an unequivocal statement about situations of a specified sort, already 'applies to the facts' in the sense that it truly describes the facts. If we are to use the formula in thought and discourse, however, we must 'apply' it in a further sense.
- (d) In our century advances that had been made, not only in pure mathematics, but also in the applying of mathematics to what happens, could not but strike many as...
- (e) A more reasonable course would be to regard geometry itself as an abstract calculus, applicable (more or less roughly) to the physical world but not descriptive of its properties.
- (f)/

- (f) For instance, a mathematical examiner criticises the performance of a candidate by applying certain mathematical principles to the work before him; when he himself is engaged in calculation, he writes down numbers in accordance with those same principles. (85)

For cases exemplified in groups (I) and (II), and for some exemplified in group (III), it is not difficult to specify activities in which items are applicatively involved. Terms, predicates, words, &c. are thus involved in what may be called 'verbal performances', e.g. referring (to), naming, describing, characterising, evaluating, signifying. Activities in which concepts, standards, criteria, and legal rules are applicatively involved may be compendiously classified as 'judgments', e.g. evaluating, appraising, assessing, deciding (a case), and judging (in an epistemological sense typified in Kant). Such activities as explaining and predicting (phenomena, events, &c.) applicatively involve physical theories, laws, hypotheses, &c. Some philosophers would claim that physical theories are applicatively involved in describing, and not just (if at all), in explaining and predicting. Philosophers have also claimed that abstract systems and their components are thus involved in describing ('facts' or 'the world'), though others would be content to say that logic and mathematics are applicatively involved only in what we may call 'reasoning about the/

the world' (including such activities as counting and measuring) and in assessing deductions and calculations for logical and mathematical correctness.

[1] Before commenting in detail on types of applicative involvement as exemplified in these sentences, however, I want first to connect the applicative mode of involvement with the instrumental and quasi-instrumental, and to develop schematically some features of a 'basic applicative formula'.

Here is a general characterisation of a 'basic applicative formula', or 'baf', in terms of α - and β -bufs.

Certain performances describable by one or another of the bufs:

'using x to δ y ',
 'using x in δ ing y ',
 ' δ ing y in accordance with x ',
 'in acting in accordance with x , δ ing y ',
 etc.,

are instances of the δ -application of x to y , and are describable by the baf: ' δ -applying x to y '.

An interesting question is why only certain bufs that exemplify one of these buf-forms can intelligibly be replaced by a corresponding baf./

baf. For instance, we do not talk of 'applying a hoe to a garden', 'applying a pen to a letter', 'applying a score to a guitar', even though the bufs:

using a hoe to dig a garden
using a pen to write a letter
playing a guitar in accordance with a score,

exemplify one of these forms. Another question concerns what we are to say about such sentences as

They might become recallable if psychoanalytical methods were applied to him,

which contain 'apply' but which do not fit one of the three groups of sentences listed above. An answer to both questions lies, I think, in the fact that none of these cases exemplifies the 'referential' aspect of applicative involvement. That is, neither hoes nor pens nor scores nor methods can be said to have reference to that to which they are 'applied' in the sense in which this can be said of words, concepts, standards, legal rules, theories, &c. This point will, I hope, be clarified in the course of the discussions which follow.

With respect to δ -application of x to y , we refer to x as ' δ -applicant (to y)', and to y as ' δ -applicate (of x)'. Bafs, like bufs, can be circumstantially specified: that is, we speak of δ -application of x to y under conditions c , or of δ -applying x to y under c . It will be recalled that we distinguish particular/

particular conditions from condition-types: this distinction is made more precise by distinguishing occasion- from type-factors. The former are exemplified in the baf ' δ -applying \underline{x} to \underline{y} at $(\underline{p}, \underline{t})$ ', where ' $(\underline{p}, \underline{t})$ ' stands for spatio-temporal co-ordinates of an occasion on which \underline{x} is (being) δ -applied to \underline{y} ; the latter is expressed in ' δ -applying \underline{x} to \underline{y} under \underline{c} ', where ' \underline{c} ' stands for a condition-type under which \underline{x} is (being) δ -applied to \underline{y} . Occasions might be regarded as instances of condition-types, whereby the baf ' δ -applying \underline{x} to \underline{y} under \underline{c} ' would mean ' δ -applying \underline{x} to \underline{y} at $(\underline{p}, \underline{t})$ & $(\underline{p}, \underline{t})$ is an instance of condition-type \underline{c} '. Whether or not this analysis is feasible, we henceforth mention only bafs specified with respect to condition-type. The question arises how these bafs are to be symbolised. In view of the relation of bafs to bufs, it might be suggested that ' δ -applying \underline{x} to \underline{y} under \underline{c} ' should be symbolised either by ' $(\underline{x} \delta \underline{y}) / \underline{c}$ ' or by ' $((\delta \underline{y}) \wedge \underline{x}) / \underline{c}$ ', depending on whether \underline{x} is an instrument or a quasi-instrument. Rather than do this, however, we adopt new symbolism to represent bafs:

' $(\underline{x} \delta \underline{y} / \underline{c})$ ' stands for the baf ' δ -applying \underline{x} to \underline{y} under \underline{c} '

A good reason for preferring to adopt new symbolism rather than to adapt the old is that correlative formulae for bafs differ in/

in sense from the R- and P-formulae defined earlier for bufs. These formulae, it will be recalled, are intended to express correlation of an instrument-method complex with a task-circumstance complex. But in defining similar formulae for applicative involvement we are interested rather in correlating applicant with applicate or, more precisely, in matching applicant and task-applicate-circumstance complex. We symbolise the sentence ' x is δ -applicable to y under c ' by the P-formula ' $P[x\delta y/c]$ ', and the sentence ' x δ -applies to y under c ' by the R-formula ' $R[x\delta y/c]$ ': the former expresses δ -applicability of x to y under c , while the latter expresses δ -validity of x for y under c . One striking difference between these correlative formulae and the others is that there seems to be no difference at all between asserting the applicability of x to y and affirming that x applies to y . If predicate P applies to x then, we want to say, P is applicable to x ; conversely, if P is applicable to x then P applies to x . Similarly for laws: to say that L applies to a case z just is to say that L can be applied to z ; to affirm the validity of a law is to imply the legitimacy of its application, and conversely, a law can be applied to a case only if it may be said to apply to it. While linguistic usage does not, apparently, warrant a sharp distinction between the senses of P- and R-formulae for/

for applicative involvement, nonetheless it is useful to keep open the possibility of such a distinction, in case a situation presents itself in which applicability and validity are plausibly distinguished.

Several types of range are associated with δ -applicability and δ -validity. (i) Unspecified ranges are defined as follows:

$$\begin{aligned} 'P(\underline{x}, \delta)' & \text{ for } '\hat{E} [(\exists \gamma) (P[\underline{x} \delta \underline{z} / \gamma])] ' \text{ (range of } \delta\text{-applicability of } \underline{x}) \\ 'R(\underline{x}, \delta)' & \text{ for } '\hat{E} [(\exists \gamma) (R[\underline{x} \delta \underline{z} / \gamma])] ' \text{ (range of } \delta\text{-validity of } \underline{x}) \end{aligned}$$

That is, ' $P(\underline{x}, \delta)$ ' stands for the set of all \underline{z} 's to which \underline{x} is δ -applicable under some conditions γ , and ' $R(\underline{x}, \delta)$ ' stands for the set of all \underline{z} 's to which \underline{x} δ -applies under some conditions γ .

(ii) There are two sorts of specified range: (a), that for which some particular condition-type is specified:

$$\begin{aligned} 'P(\underline{x}, \delta, \underline{c})' & \text{ for } '\hat{E} [P[\underline{x} \delta \underline{z} / \underline{c}]] ' \text{ (c-range of } \delta\text{-applicability of } \underline{x}) \\ 'R(\underline{x}, \delta, \underline{c})' & \text{ for } '\hat{E} [R[\underline{x} \delta \underline{z} / \underline{c}]] ' \text{ (c-range of } \delta\text{-validity of } \underline{x}); \end{aligned}$$

and (b), that for which any condition-type is specifiable:

$$\begin{aligned} 'P(\underline{x}, \delta, \underline{t})' & \text{ for } '\hat{E} [P[\underline{x} \delta \underline{z} / \underline{c}, \underline{t}]] ' \text{ (t-range of } \delta\text{-applicability of } \underline{x}) \\ 'R(\underline{x}, \delta, \underline{t})' & \text{ for } '\hat{E} [R[\underline{x} \delta \underline{z} / \underline{c}, \underline{t}]] ' \text{ (t-range of } \delta\text{-validity of } \underline{x}). \end{aligned}$$

T-ranges might also be introduced non-specifically, since the set of all \underline{z} 's to which \underline{x} is δ -applicable under tautologous conditions is identical with the set of all \underline{z} 's to which \underline{x} is δ -applicable under all conditions, and similarly for t-ranges of/

of validity. That is,

$$\hat{E} \left[\frac{P[x\delta z/c_v\bar{c}]}{R[x\delta z/c_v\bar{c}]} \right] = \hat{V} \left[(\gamma) (P[x\delta w/\gamma]) \right]$$

$$\hat{E} \left[\frac{R[x\delta z/c_v\bar{c}]}{R[x\delta z/c_v\bar{c}]} \right] = \hat{V} \left[(\gamma) (R[x\delta w/\gamma]) \right]$$

It may seem odd that, having set up this symbolic machinery, I make no use of it in the discussions which follow. My aim in schematically characterising the box ' δ -applying x to y under c ' is not, however, to design a symbolic device for use in examining concrete cases of applicative involvement: it is rather to suggest some features common to these cases, and to construct a 'picture' of this mode of involvement. This 'picture' should bring into sharp focus the referential aspect of applicative involvement, and if the symbolism does this then it will have served its purpose.

[2] Activities in which verbal expressions are applicatively involved can conveniently be distinguished into (A) reference and (B) predication. What I have in mind is illustrated by Strawson's list of "phrases... used to express [the] distinction between two complementary activities or functions involved in the complex activity of asserting... a singular proposition of a fundamental sort."

A		B
referring to something	and	describing it
naming something	"	characterising it
indicating something	"	ascribing something to it
designating something	"	predicating something of it
mentioning something	"	saying something about it

The conjunction of any A-expression with any B-expression "might serve as a description of the complex activity of making a certain sort of statement, a description which distinguishes two moments, or elements, or functions in that activity."⁽⁸⁶⁾ In terms of applicative involvement, we distinguish between the A-activity: 'rf-applying x to y ', where ' x ' stands for a referring expression and ' y ' stands for that to which x is rf-applied; and the B-activity: 'pr-applying x to y ', where ' x ' stands for a predicative expression and ' y ' stands for that to which x is pr-applied. For instance, the assertion "Socrates is a philosopher" consists of the pr-application of "(is) a philosopher" to the man to whom "Socrates" is (being) rf-applied, and the assertion "Peter plays polo" is made up of the pr-application of "plays polo" to someone to whom "Peter" is (being) rf-applied. I shall speak of pr-application both of single predicative and classificatory expressions such as "red" and "cow", as well as of verb phrases such as "(is) red" and "(is) a cow". Later I distinguish application of evaluative expressions from/

from that of non-evaluative expressions, but for the present this distinction is not important.

I want to begin my examination of rf- and pr-application by assembling some lexicographical reminders concerning usage of the verb 'apply' in this context. Then I give some instances of its employment by various logicians, and finally I address myself to the question whether, and to what extent, an instrumental 'picture' is appropriate for rf- and pr-application.

A. An early usage of 'apply' is: 'give (to a general, theoretical, or figurative statement) a specific reference to a particular instance; use it as relative or suitable to'. Among examples given in the OED are: "Which dialogue I wolde applye unto this noble prynces..." (1509) and "To apply all this to the Boeotian writers..." (1749); a modern example is: "As applied to the future, the supposition may be taken to state..." (1955). This, however, is not the usage of which rf- and pr-application are instances, though it is undoubtedly connected with the XVIIth century usage in which we are interested: 'to make use of (a word) in a special reference to, or/

or to describe or characterise (a thing)'. Some exemplifications of this usage are: "Regardant... is... only applied to a villain." (1628), "He that applied the words... to ideas different to those to which the common use applies them..." (1690), "The word fell is applied to rocky heights, peaks, and cliffs." (1877). (For modern exemplifications, see sentences in list (1) above.)

Another important usage of 'apply' as a semantic expression is its intransitive usage, which the OED cites as first appearing in the XVIIIth century: 'to have practical bearing upon, a valid or suitable reference to'. Only the second part of this definition is pertinent here. It is nowadays an extremely common usage, as typified in these sentences:

- (a) Those names (or descriptions) which don't apply to anything are all names of the same thing...
- (b) A unique description is presumably one which applies only to a single entity...
- (c) To say that an expression applies to certain occasions is just to say that it has a descriptive meaning; (87)

Finally, there is the adjective 'applicable', which the OED misleadingly defines by 'capable of being applied; having reference'. It seems to me that 'capable' is not right here, for it fails to convey the idea of propriety or suitability or appropriateness of application that is normally implicit in 'applicable'. If 'capable of being applied' is understood as/

as equivalent in force to 'legitimately applied' or 'correctly applied', then I have no quarrel with the OED. As to the second half of this definition, it is evident that 'x is applicable to y' has the same sense as 'x applies to y', if these are understood to be equivalent to 'x has (suitable) reference to y'. As we shall see later, this ambiguity between transitive and intransitive usages of 'apply' leads to some interesting logical problems. (The possibility of confusing 'x applies to y' and 'x can be applied to y' appears to be a mark of the concept of application.)

B. . . Locke makes interesting use of 'apply' in Book III of the Essay. He there adopts an extreme version of the view that meanings are ideas or concepts, and, indeed, seems to hold that in using words (voces) meaningfully one applies them to his ideas. My speech is understood, according to Locke, "when, by use or consent, the sound I make by the organs of speech excites in another man's mind the idea I apply it to in mine when I speak it."⁽⁸⁸⁾ To the question "To what it is that names, in the use of language, are immediately applied,"⁽⁸⁹⁾ Locke replies: they are immediately applied to the speaker's ideas.

That/

That then which words are the marks of are the ideas of the speaker: nor can anyone apply them as marks, immediately, to anything else but the ideas that he himself hath: (90)

Only if the speaker as it were attaches his utterance to the same ideas as those excited in the mind of a hearer, is 'the end of speech... that these sounds, as marks, may make known his ideas to the hearer' attained. In their role as 'immediately the sign of men's ideas' words serve as "the instruments whereby men communicate their conceptions." (91) In the light of this, we hazard the following guess as to why Locke talks of applying words, or sounds, to ideas: similarly as we might view 'xf-applying x to y ' as equivalent to 'using x to refer to y ', so we might take 'using x to signify y ' (or: 'using x as a sign of y ') to be equivalent to 'sg-applying x to y ', where ' y ' stands for an idea which the word x is 'made to stand for or signify'. But 'to signify' is not at all the same verb as 'to refer (to)', and 'sg-application' is not at all like 'xf-application': it is apparent that Locke fails to distinguish between them; only with Mill's (revival of a) denotation/connotation distinction is the importance of this point (again) grasped.

Both Mill and De Morgan exploit 'apply' as a semantic expression. The former writes, for example, "all names which can be said to have any signification, by applying which to an individual/

individual we give any information respecting that individual, may be said to imply an attribute of some sort." (92) Since Mill is willing to say that we predicate proper names of their bearers, he can be said to equate 'applying x to y' with 'predicating x of y'. Thus, the expressions "Sophroniscus", "the father of Socrates", "a man", "a Greek", &c. are applicable to Sophroniscus, though the first is applied to him "merely to distinguish him from other persons who are spoken of" whereas the others are applied to him "to indicate a fact relating to him." (93) Despite Mill's distinction between connotative and non-connotative terms, he fails to distinguish between the modi applicandi of terms: in asserting "Sophroniscus is a Greek", one does not predicate "Sophroniscus" of the individual of whom he predicates "a Greek"; rather, one re-applies "Sophroniscus" to someone to whom he pre-applies "(is) a Greek". Nor does De Morgan draw such a distinction when he makes 'applicable' the basic semantic expression of his 'onymatic system'. He states that "the only relation of a term to a thing is that of applicable or not applicable" and that "the relations between terms, the only ones admissible because they are terms... are those of applicable to some the same object and not applicable to any the same object." (94) In De Morgan's onymatic system, terms and their complements are conjoined by 'have joint/

joint application' and 'have no joint application' into "purely onymatic enunciations" such as 'X and \bar{Y} have no joint application' (i.e. Every X is Y) and ' \bar{X} and Y have joint application' (i.e. Some Y's are not X's).

The suggestion is made, in an article written for Baldwin's Dictionary by Peirce and (in part) Mrs Ladd-Franklin, that 'application' and 'signification' be used in place of Mill's 'denotation' and 'connotation'.⁽⁹⁵⁾ An interesting ground for this suggestion is that "these words may be applied to the corresponding properties of propositions as well as terms." Thus, the application of a term is "the collection of objects which it refers to" and its signification is "all the qualities which are indicated by it", while the application of a proposition is "all the instances of its holding good" and its signification is "all its different implications". Later, however, the 'complete application (or range)' of a proposition is defined as "all those descriptions of circumstances under which it holds good -- that is to say, all its sufficient antecedents." It is important to note the difference between these two definitions of the 'application' of a proposition. Take, for example, the proposition that John is a son of Henry: on the first definition, its 'application' consists of sets of individual-pairs (x, y) such that "John" is rf-applicable to/

to x , "Henry" is rf-applicable to y , and x stands in the relation "... is a son of..." to y ; on the second definition, however, its 'application' consists of descriptions of conditions under which it can truly be asserted that John is a son of Henry. In other words, if, for individual-pair (x,y) , "John" is rf-applicable to x and "Henry" is rf-applicable to y ; and if for some y to which "Bill" is rf-applicable, x stands in the relation "... is a brother of..." to y and y stands in the relation "... is a son of..." to z ; then x stands in the relation "... is a son of..." to z and (x,y) falls within the 'application' (in the first sense) of the proposition that John is a son of Henry. (Instead of saying ' x stands in the relation "...R..." to y ' we could also have said '"...Ry" is pr-applicable to x ' or '"xR..." is pr-applicable to y '.)

Frege writes, in 'On Sense and Reference':

A proper name (word, sign, sign-combination, expression) expresses its sense, stands for or designates its reference. By means of a sign we express its sense and designate its reference. (96)

Frege's use of 'express' and 'designate' in the first sentence exemplifies an 'intransitive' usage of semantic expressions; his use of these words in the second sentence exemplifies a 'transitive' usage. Many semantic expressions have both transitive and intransitive usages, as this list indicates.

<u>Transitive</u>	<u>Intransitive</u>
<p>"A" is used to {</p> <ul style="list-style-type: none"> refer to \underline{x} name \underline{x} designate \underline{x} denote \underline{x} <p>"B" is used to {</p> <ul style="list-style-type: none"> characterise \underline{x} describe \underline{x} signify \underline{y} express \underline{y} connote \underline{y} 	<p>"A" {</p> <ul style="list-style-type: none"> refers to \underline{x} names \underline{x} designates \underline{x} denotes \underline{x} <p>"B" {</p> <ul style="list-style-type: none"> characterises \underline{x} describes \underline{x} signifies \underline{y} expresses \underline{y} connotes \underline{y}

It might be argued that intransitive usages of semantic expressions collapse into transitive, since it could not be said of an expression "A" that it δ \underline{x} unless "A" is (correctly) used to δ \underline{x} . It may further be suggested that logicians have traditionally followed intransitive usages merely on grounds of simplicity, and that they would, if pressed, assent to substitution of "A" is (can be) used to δ \underline{x} ' for their intransitive locution "A" δ \underline{x} '. Some contemporary logicians, for example Strawson, might readily endorse this suggestion, acknowledging that their intransitive locutions are ellipses for transitive. I do not venture to guess whether other logicians would accept such a proposal, though later I mention grounds on which the non-collapseability of intransitive into transitive locutions might be defended. (97)

While/

While there is little danger of confusing transitive and intransitive usages of the above-listed semantic expressions, it is otherwise for the compendious expressions 'apply' and 'predicate'. The reason for this is, as I see it, that an expression of instrumentality, which is characteristic of transitive semantic locutions (e.g. '"A" is (correctly) used to δ \underline{x} ', ' \underline{x} is (correctly) δ ed by (means of) "A"'), is external to the above-listed verbs, whereas it is, as it were, built into 'apply' and 'predicate'. In consequence, a transitive/intransitive distinction is not mirrored in the grammar of these verbs, and the two usages are apt to be confused. '"A" is predicable of \underline{x} ' (i.e. '"A" is correctly predicated of \underline{x} ' or '"A" can be predicated of \underline{x} ') and '"A" is a predicate of \underline{x} ' are, like '"A" is applicable to \underline{x} ' and '"A" applies to \underline{x} ', apparently equivalent: for it seems that if "A" can be predicated of \underline{x} then "A" is a predicate of \underline{x} , and vice versa. (This is like saying 'This is red' and 'This can be described as "red"' are equivalent.) It will perhaps be objected that to call an instance of this ambiguity a 'confusion' is to make a mountain out of a mole hill. To the extent that transitive and intransitive locutions are interchangeable, and in so far as the latter collapse into the former, this objection must be granted. It may, on the other hand, be of some interest to examine a couple of instances of the 'confusion', and/

and to enquire into the plausibility of maintaining a sharp transitive/intransitive distinction.

A good example occurs in C.I. Lewis's definitions, in terms of 'apply', of 'denotation' and 'term'. Lewis first defines the 'denotation' of a term as "the class of all actual things to which the term applies", but he later calls it "the class of all actual or existent things which the term correctly applies to or names."⁽⁹⁸⁾ I am puzzled by the word 'correctly' in this definition: it seems to me that a term may be correctly or incorrectly applied to something, but that it does not itself correctly or incorrectly apply to anything. Just as I find 'correctly' puzzling in this context, so the word 'capable' perplexes me as it occurs in Lewis's definition of 'term' as "an expression capable of applying to a thing or things of some kind."⁽⁹⁹⁾ It will be recalled that the OED defines 'applicable' by 'capable of being applied', and that, as I suggest above, 'capable' is misleading here unless understood to have the force 'correctly'. But 'capable' in the intransitive locution employed by Lewis, 'capable of applying to...', seems entirely out of place, though perhaps in a secondary sense 'capable' can be used in similar contexts. If, for example, we talk of the 'jobs' or 'functions' /

'functions' of expressions, then to describe "A" as 'capable of applying to' (or: ... of naming, ... of referring to, ... of describing, ... of expressing, &c.) such-and-such, is to mention one of the 'jobs' which "A" can (be used to) do -- somewhat as describing a type of steam hammer as 'capable of driving pilings into clay' amounts to specifying a 'job' which it can (be used to) do. (100) But this secondary sense must itself be explicated in terms of a transitive locution, and when a logician says 'capable of applying to' or 'can apply to', he must surely be understood to mean 'capable of being applied to', and hence 'applicable to' or 'correctly applied to' or 'would be correctly applied to'. (100a) It is interesting to observe that Lewis employs -- ostensibly as synonymous semantic expressions -- the following variations on 'apply': 'is applicable to', 'would be correctly applicable to' (sic!), 'would correctly apply to', 'is capable of applying to', 'correctly applies to', and simply 'applies to'.

Quinton, in a recent essay, introduces 're-applicability' to explain the function of general terms. His use of 'apply', however, exemplifies the same confusions that we find in Lewis; it also hints at a defence against the view that intransitive are collapsible into transitive semantic locutions. Quinton begins by stating that a 'predicatively used general term' has "a class as/

as its extension, the range of its correct application"; a few lines later, though, he asserts that 'man' and 'featherless biped' are terms which "apply to just the same things." (101) This suggests that a general term 'applies to' items which fall within its 'range of correct application': but the range of correct application of "A" must surely be the class of objects to which "A" is correctly applied, or to which "A" can be applied. In the following passage Quinton introduces 're-applicability'.

A general word is one that applies to a multiplicity of things and the special, problematic feature of general words is their re-applicability. This means that if someone is shown a few instances of the application of such a word, he can go on, without hesitation and in agreement with the practice of others, to identify other instances. (102)

He returns, two pages later, to elucidate this notion:

What creates the problem is simply re-applicability, the fact that predicative terms can apply to an indefinitely large number of individual things. (Not, of course, that they always or necessarily do.) How is it that we can attach meaning to and make use of words which apply to things we have never encountered before? We ordinarily do not, and indeed very seldom can, learn how to use them by being introduced to the whole range of their application. Even if we have in fact met the whole range of application, our knowledge that we have done so is likely to be pretty insecure. (103)

Note that Quinton here refers to 'range of application' and not, as before, to 'range of correct application': this is better, for 'range of application of "A"' means simply the class of things to/

to which "A" applies. Note, too, his phrase "... terms can apply to...": it is not clear to me just how this 'can' connects with the modal force of 're-applicable', unless Quinton wants to say '... terms can be applied to...'. On the other hand, if I understand his argument rightly, Quinton would not agree to substitution of his intransitive usage of 'apply' by the transitive usage. What he calls a 'natural class' contains items to which a predicative general term applies, whether or not they have as yet been encountered and identified as members of the class. Because "A" pr-applies to x - i.e. because x is a member of the natural class C1"A" - it can be said that "A" is pr-applicable to x, or that "A" is correctly pr-applied to x. But whether "A" pr-applies to x does not depend on the pr-applicability of "A" to x, since x is (is not) a member of C1"A" whether or not it has been identified as a member of that class. The fact stated in '"A" pr-applies to x' is, as it were, a fact 'about' the world, whereas the fact that "A" is pr-applicable to x is a fact 'about' the expression "A" rather than 'about' x. This is a crucial point, and difficult to express coherently. To say that "A" is pr-applicable to x is to say that, usage of "A" being what it is, it is (would be) correct to pr-apply "A" to x; but to say that "A" pr-applies to x is to say that, the world being/

being as it is, x is a member of the natural class $Cl"A$.

On the other hand, if x is a member of $Cl"A$ then it is (would be) correct to pr-apply "A" to x , and conversely, if "A" is (would be) correctly pr-applied to x then x is a member of $Cl"A$.

In spite of this apparent equivalence, however, we say something different in asserting that "A" pr-applies to x and in claiming pr-applicability of "A" to x : for we might deny that there are natural classes such as $Cl"A$, or deny their necessity for explaining the pr-applicability of general terms to an indefinitely large number of things. Since Quinton believes both that there are such classes and that they are necessary for this explanation, (10) he might want to distinguish between assertion of membership in $Cl"A$ and assertion of correctness of application of "A", and this amounts to a distinction between 'A' pr-applies to x ' and 'A' is pr-applicable to x '.

C. I want now to examine the view that rf- and pr-applied expressions are instrumentally involved in performances of the 'tasks' of referring (to) and describing. We find Strawson explicitly characterising reference and predication in terms of 'tasks':

The/

The task of forestalling the first question [sc. 'What (who, which one) are you talking about?'] is the referring (or identifying) task. The task of forestalling the second [sc. 'What are you saying about it (him, her)?'] is the attributive (or descriptive or classificatory or ascriptive) task. (105)

But he later says: "to use a separate expression to perform the first of these tasks is to use an expression in the uniquely referring way." (106) We are apparently to say that 'referring to x' has to do both with the performance of a task in which an expression is instrumentally involved and, at the same time, with the way in which this expression enters into discourse. Thus, the baf 'rf-applying "A" to x' is construable either as the α -baf 'using "A" to refer to x' or as the phrase 'referentially applying "A" to x': Strawson's talk about using expressions 'referringly' and his identification of the 'uniquely referring use' with 'a way of using expressions' are reflected in the latter interpretation of rf-application, while his talk about the 'task of referring' fits the former. Wittgenstein, in The Blue Book, also muddles the 'task' and 'way' terminologies. He says, for example,

When something seems queer about the grammar of our words, it is because we are alternatively tempted to use a word in several different ways. And it is particularly difficult to discover that an assertion which the metaphysician makes expresses discontentment with our grammar when the words of this assertion can also be used to state a fact of experience. (107)

Again, Wittgenstein asks whether "the person who talks both of conscious and unconscious thoughts thereby uses the word 'thoughts' in two different ways"; (108) but in subsequently comparing 'ways of using a word' with 'ways of using a hammer', he seems to be talking rather about tasks in performance of which a hammer is instrumentally involved.

Conflation of 'way of using "A"' with 'task for which "A" is used' is considered in two recent articles in Analysis. In the first, A.R. White distinguishes between 'how' and 'what-for' questions:

Now I believe that, although we can certainly and importantly distinguish the "how" question, that is, "the way, method or manner" in which an expression is used, and the "what-for" question, that is, the purpose for which an expression is used, these are both parts of the use of the expression. (109)

In a discussion of White's distinction, Caton differentiates between two types of 'what-for' question, the first type represented schematically in 'What are X's used for?', and the second in 'What did NN use that X for?'. (110) He claims, however, that type-one 'what-for' questions constitute a species of the 'how' question schematised in 'How are X's used?', for the case in which X is an expression-type (if not in all cases).

Questions/

Questions about the use of an expression... are about things that are regularly done by using the expression ... Thus the same kind of question about an expression E can be raised by asking 'what is E used for?', 'what is the function of E?', 'how is E used?', 'how do we use E?', or 'what are the uses of E?'. (This shows, incidentally, that the distinction on which White says he agrees with Ryle should be between 'how' questions and type-two 'what-for' questions, type-one 'what-for' questions being one variety of 'how' question.) (111)

Caton lists 'referring' and 'describing' among 'activities which are normally conducted by using linguistic expressions', along with 'expressing our feelings, asserting, questioning, promising, swearing oaths, etc.' He also says that type-one 'what-for' questions, "when asked about linguistic expressions, are closely similar to the same type of question asked about non-linguistic things." (112) It follows, then, that the question 'What is "the king of France" used for?' is like the question 'What are hammers used for?', in that by answering 'For referring to a man who...' to the first, and 'For driving nails...' to the second, we mention a function of the expression(-type) "the king of France" and of hammers. But if, as Caton claims, the point of asking 'What is "the king of France" used for?' is subsumed under that of asking 'How is "the king of France" used?', then we should expect the point of 'How are hammers used?' to be closely related to that of 'What are hammers used for?'.

Our/

Our expectations are, naturally enough, disappointed. The 'how' question asked of hammers is incomplete as it stands, for unless we have in mind a task-type or particular task, the question 'How are hammers used?' is nonsense: 'How are hammers used for doing such-and-such?', on the other hand, makes perfectly good sense. But the 'how' question asked of expression-types is not, in this way, incomplete: indeed, the question 'How is "the king of France" used for referring to a man who...?' is as odd as the question 'How are hammers used?'. On the other hand, both the question 'How do people refer to a man who...?' and the question 'How do people drive nails?' are intelligible; as are the questions 'What do people use for referring to a man who...?' and 'What do people use for driving nails?'; but these are obviously different questions from the others. Somebody might know that hammers are (can be) used to drive nails without knowing how to use a hammer to drive a nail, and in learning how to use a hammer to do such a task he acquires technique for the exploitation of a capacity: this technique is correlated with a method, which may or may not be an optimum method for using that type of hammer to do that type of task. On the other hand, it is difficult to conceive of someone knowing that an expression-type is (can be) used to refer to a man who... without/

without knowing how to use such an expression to refer to such a man: this is, as I see it, because the idea of coming to know how, or of learning how, to use an expression to refer to something simply fails to 'get a grip'. Are there methods and techniques of rf-applying an expression to something, similarly as there are methods and techniques of using a hammer to drive nails? The idea is ludicrous. "But," it will be objected, "surely it's possible to go wrong in referring to something, to rf-apply an expression to something to which it is rf-inapplicable. As Strawson says, the meaning of a (referential) expression consists in 'rules, habits, conventions governing its correct use, on all occasions, to refer', and to give its meaning is to give 'general directions for its use to refer to or mention particular objects or persons', (113) someone who misapplies these directions is in the position of one who fails to observe instructions in using a device; he goes wrong by not using the (verbal) device in accordance with the proper - and hence optimum - method or technique. And Evans suggests that 'we may regard words as the tools which we use to do a certain job', and that:

The tools can be badly handled and hence fail to do the intended job, but in that case we should not describe the tools as improper but rather the using of them." (114)

Against/

Against this objection I would urge, in the first place, that the 'picture' associated with (manipulative) instrumentality, as it is developed in this paper, does not support an analogy between words and tools. Conventions which circumscribe correct pr- or rf-application of expressions are not like methods or techniques for efficient use of a tool: the question of 'efficiency' in the case of words and expressions collapses into the question of 'correctness' or 'propriety', and this latter question does not arise in the case of instruments and tools (though it might!). (For some further objections to the instrumental analogy, I refer the reader to articles by L.J. Cohen and C.K. Grant.)⁽¹¹⁵⁾ Though the device analogy - comparing words and expressions to (non-manipulated) devices - is less objectionable, again it seems to me that linguistic conventions differ in important respects from instructions for the 'correct' operation of a device. Finally, it may perhaps be helpful to think of linguistic conventions rather as C-rules of linguistic practices than as R-rules in accordance with which verbal performances are carried out. That is, we might say: by following conventions of rf-application of "A", we correctly rf-apply "A"; rather than: if we rf-apply "A" in accordance with conventions of rf-application of "A", then we correctly rf-apply "A".

In the preceding discussion I have not scrupulously taken account of the 'type/token' distinction. In general I have argued that expression-types are not appropriately assimilated to instruments, and it might be suggested that this analogy is felicitous in the case of expression-instances. In this connection it is interesting to read what the psychologist Skinner says about 'using words':

...although the formal properties of the records of utterances are interesting, we must preserve the distinction between an activity and its traces. In particular we must avoid the unnatural formulation of verbal behaviour as the "use of words". We have no more reason to say that a man "uses the word water" in asking for a drink than to say that he "uses a reach" in taking the offered glass. (116)

(I take it that by 'the word water' Skinner means 'an utterance water' or 'an expression-instance water', for comparison with 'a reach' would be odd if he meant 'the word-type water'.) L.J. Cohen also criticises the phrase 'using an expression-instance':

The use of a word-instance just is its sole and single occurrence, and we cannot therefore regard the verb 'to use' when we speak of 'using a word-instance' as functioning in the same logico-linguistic category as when we speak of 'using a tool'. (117)

But the OED gives a sense of 'to use' in which 'using "A"' means 'speaking or writing "A"' or, in other words, simply 'uttering "A"', where/

where "A" presumably stands for an expression-type. It follows that we can regard the phrase 'using "A" to refer to x' as a description of an utterance of "A" which, in view of circumstances and conventions of re-application of "A", is understood to constitute reference to x. That is to say, NN refers to x in uttering "A" if, and only if, his utterance to some degree satisfies these circumstantial and conventional criteria. Note that we also say: NN's utterance of "A" refers to x. And that we say: "A" refers to x. But we can say this only because of the circumstantial and conventional criteria whose satisfaction by someone's utterance of "A" suffices to make him understood to be referring to x. Saying of a word that it refers to (describes, characterises, names, &c.) something is like pointing to the heart in an anatomist's drawing and saying "This pumps blood."

[3] Turning now to the type of applicative involvement exemplified in sentences of group (II), involvement of concepts, criteria, standards, and legal rules in activities compendiously called 'judgments', I first consider two theories of concept-application/

application and then deal with some questions concerning application of standards and legal rules.

A. Geach, in his recent book Mental Acts, holds that concepts are applicatively involved in 'acts of judgment': he explicitly denies that 'concepts are somehow exercised singly without being applied to anything'. Concepts are, for Geach, mental capacities 'exercised in acts of judgment'. The relation of concepts to an act of judgment in which they are exercised (i.e. in which they are applicatively involved) resembles the relation between "a number of simple, previously acquired, skills" and an appropriate chess-move in which these skills are exercised.⁽¹¹⁸⁾ Geach employs another chess analogy to explain his denial that concepts can be 'exercised singly without being applied to anything':

... in chess one can practice the moves of a solitary man on the board, but I think there is no analogous exercise of a single unapplied concept.⁽¹¹⁹⁾

This analogy might be misleading, however, for it seems to deny the existence of acts of judgment in which only one concept is applicatively involved, whereas I believe Geach means to claim only that no matter how many concepts are involved in an act of judgment, they must be applicatively involved, i.e. concepts are 'applied to' and not simply 'exercised'. (On the other hand, Geach may want to emphasise 'single', rather than 'unapplied', in his/

his phrase 'single unapplied concept'.)

Geach talks of applying concepts to, inter alia, 'the material environment', 'physical things', 'things', 'human beings and their natural environment', 'an earthworm', and 'a supposed disembodied existence'. While he says that a sighted man is able to apply his colour-concepts to 'visual experience', (120) Geach nonetheless denies that concepts are fundamentally or primarily applied to sense-experiences or sensations. Involvement of 'sensory' concepts in acts of judgment about sensations is, he claims, "secondary to the application of 'sensory' concepts to the material environment." (121) In arguing against the view that concepts are formed by 'abstraction' from experience, he asserts that a non-experiential basis of concepts "does not in the least prevent us from applying concepts in our sense experience and knowing sometimes that we apply them rightly. In all cases it is a matter of fitting a concept to my experience..." (122) He repeats this 'fitting' metaphor later:

In all cases it is a matter of fitting my concept to my experience - of exercising the appropriate concept - not of picking out the feature I am interested in from among others simultaneously given in experience. (123)

But data of experience are not themselves involved in acts of judgment:

The/

The content of the judgment is always intelligible and conceptual - acquaintance with a particular sensible thing is no part of the judgment itself - but an act of judgment performed in a particular sensory context may thereby be related to particular sensible things. (124)

Hence, we should not speak of non-conceptual factors as involved in acts of judgment, for the content of such acts consists solely of exercises of concepts and relations among these exercises. For example, judgments expressible in the assertion "That flash was before this bang" have the same 'intelligible content', namely an exercise of the concept flash standing in a relation §(before) to an exercise of the concept bang; but individual judgments stand in a special relation to particular 'sensory contexts', and Geach warns "it is a great mistake to try to bring in these contexts into any setting forth of that which is judged." (125) Hence, objects to which concepts have application - whether primary ('the material environment') or secondary ('sense-experiences') - are necessarily external to judgments about them. Consider a judgment expressible as "The cat is eating the liver", made in a particular 'sensory context' of seeing a cat eating liver: according to Geach, the concept cat eating liver, as exercised in this judgment, is applied to a cat eating liver (or: to a cat and a piece of liver) in/

in so far as seeing the cat eating the liver constitutes the 'sensory context' of this judgment. This, at least, is my interpretation of these remarks of Geach:

Now when I make this judgment about the cat and the liver, it is not the cat and the liver that are parts of my sensory experience, but my seeing the cat and the liver. The concept of cat eating liver is exercised, then, in a context of sensation; my judgment is a judgment about the cat and the liver because I see them. (126)

It can be seen that I take 'judgment J is about object x' to be equivalent to 'some concept C, exercised in J, is applied to x'. This interpretation seems to be supported by another of Geach's examples. A person who says, while looking at the moon, "This looks like a yellow disc one foot across" is said to have the thought (of) a yellow disc a foot across: "the concepts exercised in this thought are primarily applied to physical objects; and what gives his exercise of the concepts a particular application in a certain bit of his sense-experience - of his visual sensations." (127) Note that Geach talks here of the exercise of concepts as having 'a particular application': this is odd, for he has heretofore talked only of concepts themselves in terms of 'application', and has intimated that to exercise concept C is to apply C to something. Leaving this quibble aside, however, Geach's point in the present example is that concepts acquired "in their application to the physical things around us" are analogically exercised in a judgment expressible/

expressible as "This looks like a yellow disc one foot across", in so far as they serve here "to describe the sensation itself." (128) That is to say, in a sensory context constituted by seeing a yellow disc a foot across (i.e. constituted by these visual sensations), concepts exercised in a judgment expressible as "This is a yellow disc a foot across" are therein applied to the (seen) yellow disc; whereas in a sensory context constituted by seeing the moon (i.e. by these visual sensations), concepts exercised in a judgment expressible as "This looks like a yellow disc a foot across" are therein applied to visual sensations constitutive of the sensory context, this application, however, being 'transferred or analogical' with respect to their primary application to a (seen) yellow disc (i.e. with respect to their exercise in a sensory context constituted by seeing a yellow disc a foot across).

In contradistinction to Geach's view that concepts are primarily applied to 'physical objects' and only secondarily applied to 'sense-experiences' stands Kant's view of concept-application, at least as he states it here:

Since no representation, save when it is an intuition, applies directly to an object, no concept is ever directly referred to an object, but to some other representation of it, be that other representation an intuition, or itself a concept. (129)

It/

It should be noted that I depart from Kemp Smith's translation in rendering 'auf... gehen' by 'apply... to' rather than by 'is in relation... to' and, more importantly, in rendering 'wird... auf... bezogen' by 'is... referred to' rather than by 'is... related to'. My reason for these translations is that they emphasise a distinction between intransitive as against transitive concept-applicate relations. This contrast is repeated in two other sentences from the same page of Kritik. First, 'werden bezogen', expressive of a transitive relation, is set against 'gelten', which expresses an intransitive:

- (a) In every judgment there is a concept which holds of many representations, and among them of a given representation that is directly referred to an object. (130)

(Strictly speaking, Kant does not here contrast 'gelten' ('holds of') and 'werden bezogen' ('is... referred to') with respect to concept-applicate relations, since the 'representation that is directly referred to an object' is an intuition. Nonetheless, in the two passages so far quoted Kant has employed transitive and intransitive locutions with both types of 'representation', concepts and intuitions.) Immediately following this sentence, Kant writes:

(b)/

- (b) Thus in the judgment 'all bodies are divisible', the concept of divisibility is related to various other concepts, but is here referred in particular to the concept of body, and this concept again to certain appearances that present themselves to us. (130)

Here the intransitive relation expressed in 'bezieht sich... auf' ('is related to') is contrasted with the transitive relation expressed, again, in 'wird... auf... bezogen' ('is... referred... to'). Kemp Smith's translation of the latter by 'is... applied ... to' is perhaps preferable to 'is... referred... to', but I want to reserve 'apply' for rendering 'anwenden', which Kant employs to express the transitive relation between pure concepts (categories) and appearances or 'objects of experience'.

The heading of §22. of the Second Edition transcendental deduction is:

The Category has no other employment in knowledge of things than its application to objects of experience; (131)

and that of §24. is:

The application of the categories to objects of the senses in general. (132)

At one point in §22. Kant concludes "the pure concepts of understanding, even when they are applied to a priori intuitions, as in mathematics, yield knowledge only in so far as these intuitions - and therefore indirectly by their means the pure concepts/

concepts also - can be applied to empirical intuitions." (133)

Here we find Kant talking of intuitions, as well as concepts, in terms of 'anwenden'. Both are, as we have seen, representations (Vorstellungen) which stand to applicates in both transitive and intransitive relations: this reflects the two aspects of applicative involvement, instrumentality (or, in the case of Kant's Begriff, quasi-instrumentality) and referentiality.

Representations are involved in mental (cognitive) acts, but they also have reference to 'objects': that is to say, they are applied to objects in cognitive acts. "The concept of body, " Kant writes, "means (bedeutet) something, for instance metal, which can be known by means of (durch) that concept." (134) We use the concept of body to know that which the concept means; this can be put also: 'We apply the concept of body to that to which it applies.' It is interesting to note that Kant introduces the problem of a transcendental deduction both in terms of justifying "the pure a priori employment" of certain concepts among those which "form the complicated web of human knowledge," and also in terms of explaining "the manner in which concepts can thus relate a priori to objects." (135) Again, he brackets suspicion as to 'the objective validity' of pure concepts together with suspicion as to 'the limits of their own employment'. (136) In sum, Kant's Begrifflehre is pervaded by that tension between instrumentality and/

and referentiality which, I have suggested, characterises instances of the concept of application.

This comes out strikingly in his doctrine of transcendental schematism. Transcendental philosophy is said to have "the peculiarity that besides the rule (or rather the universal condition of rules), which is given in the pure concept of understanding, it can also specify a priori the instance to which the rule is to be applied." (137) But the chapter on Schematism treats of "the sensible condition under which alone pure concepts of the understanding can be employed." (138) Note the difference, incidentally, between specifying 'the instance to which the rule is to be applied', as against specifying 'the conditions under which alone the rule can be employed!'. Kant reformulates his problem in terms of the latter:

How, then is the subsumption of intuitions under pure concepts, thereby the application of a category to appearances, possible?... We must be able to show how pure concepts can be applied to appearances. (139)

But he then proceeds to characterise transcendental schemata in terms both of (a) conditions under which categories can be applied, and of (b) 'mediating representations' which make application of categories to appearances possible.

(a)/

- (a) The schemata of the pure concepts of understanding are thus the true and sole conditions under which these obtain relation (Beziehung) to objects and so possess significance (Bedeutung).
- (b) The schema is, properly, only the phenomenon, or sensible concept, of an object in agreement (Übereinstimmung) with the category. (140)

Here, as so often in reading Kant, I fail to comprehend, I can only remark the difference between saying (a) 'Unless x is given in conditions S, then concept C cannot be applied to x', and saying (b) 'If x is given through representation R, then concept C can be applied to x'. I would, if space and comprehension permitted, enquire both into the difference between (a) and (b) and into the nature of the 'object' to which Kant's concepts and intuitions are related: this would, however, involve lengthy exegesis of such topics as 'synthesis', an undertaking into which I feel ill-equipped to venture.

Before turning to consider application of standards and legal rules, I want to mention two further points concerning concept-application. (1) It is interesting that Kant is the first writer on epistemology to talk extensively in terms of 'using' or 'applying' concepts: in general, empirical philosophers had spoken of the mind or understanding as operating on, rather than with, ideas or concepts. There are, of course, notable exceptions, /

exceptions, e.g. in Berkeley's and Hume's discussions of general words and ideas.⁽¹⁴¹⁾ (ii) While Geach regards concepts dispositionally (as capacities exercised in acts of judgment), Kant views concepts in terms of a legal analogy (as rules under which objects are subsumed): it is also plausible, however, to compare concept-application with the application of predicative expressions, i.e. to correlate concepts with pre-applied words. This analogy is exploited by Körner in his book Conceptual Thinking, where he defines 'concept' in terms of 'sign' as follows:

By adding a synonymity rule to the rules which govern the use of a sign as a predicate we again change its use. The sign is then being used no longer as a predicate but as a concept.⁽¹⁴²⁾

"Körner, it may be added, conducts his enquiry in terms of 'applying concepts to bases'.

B. In dealing with application of standards, I distinguish between ev- and nev-application: the baf 'ev-applying S to x ' is equivalent to the β -buf 'evaluating x in accordance with (by reference to) standard S', and the baf 'nev-applying S to x ' is equivalent to the β -buf '(non-evaluatively) classifying x in accordance with standard S'. What I call 'ranking standards', whether ev- or nev-applied, are formulated in hypothetical sentences of the form:

(S)/

(S) "y" is applicable to x, on condition that x is adjudged to satisfy criteria (A,B,C) to degree n.

What I call 'non-ranking standards', on the other hand, are formulated in hypothetical sentences which stipulate merely that a certain expression is applicable to anything adjudged to satisfy certain criteria. While evaluations ordinarily involve ranking standards, non-evaluative classifications may involve either ranking or non-ranking standards. For instance, classifying a horse as to size consists in nev-applying a ranking standard, while applicability of 'female' to a horse is decided through nev-application of a non-ranking standard; evaluating a horse, on the other hand, invariably is done by ev-applying a ranking standard. It will be convenient to distinguish predication - pr-application - from value-predication - vpr-application. Pr- and vpr-application are modes of expression-application associated with ev- and nev-application of standards: pr-applicability is determined by nev-application, and vpr-applicability is decided through ev-application of a standard. (I do not mean to suggest that pr-application must be associated with nev-application of a standard, for obviously there are many expressions whose pr-applicability is determined without nev-application of any standard; on/

on the other hand, it seems to me that there are no expressions whose vpr-applicability is determined without ev-application of some standard. This also raises the question, whether 'evaluating' names a speech-act or a 'mental act': the answer is, of course, that it can be used to refer either to a speech-act ('vpr-application') or to a 'mental act' ('ev-application').)

The logic of ev- or nev-application of standards is quite simple. To apply S to \underline{x} , where S is a ranking standard, is to determine the degree to which \underline{x} satisfies the criteria of S, $C(S)$: the conjunction of S and a statement that \underline{x} satisfies $C(S)$ to degree \underline{m} entails, by modus ponens, applicability to \underline{x} of some expression " \underline{Y}_m ". Similarly for the case where S is a non-ranking standard. On this level, at least, there is apparently no difference between ev- and nev-application. We must go outside logic to find a relevant point of differentiation. One such point lies in the distinction between pr- and vpr-application: as Urnson says, "to describe is to describe, to grade is to grade, and to express one's feelings is to express one's feelings, and none of these is reducible to either of the others." (143) Since pr-applicability follows from nev-application, and vpr-applicability follows from ev-application, we can distinguish between modes of standard-application in terms of a distinction between modes of expression-application./

expression-application. Another point is aptly stated by Baier, when he says that in ev-application "we are concerned not merely with the nature of the thing in question, but also with how well a thing of this nature can minister to our wants, desires, aims, needs, aspirations, ideals, and the like." (144)

The interest in, or purpose served by, ev-application is practical in the sense that it connects with giving advice concerning courses of action. This is not to say that ev-application issues in commendation, for vpr-application of expressions is no more a commendatory use of speech than is pr-application of expressions: to vpr-apply 'good' is to record the outcome of ev-applying some standard, and though someone may take it as a commendation it need not be meant to be more than a value-predication - it should be remembered, incidentally, that pr-application of 'big' in a certain tone of voice might sometimes be understood to be a commendation. A final point at which ev- and nev-application are distinguishable is that of the relative challengeability of ev-applied as against nev-applied standards. Suppose both S, which embodies criteria (A,B,C), and T, which embodies criteria (D,E,F), to be conjointly nev-applicable to an object, such that nev-application of S entails pr-applicability of an expression that is antithetical in sense to an expression whose pr-applicability is entailed/

entailed by nev-application of T: in such a case we would normally remark simply that S and T are, after all, different standards. But imagine an analogous disparity in the case of two conjointly ev-applicable standards S (whose ev-application entails vpr-applicability of, say, 'good') and T (whose ev-application entails vpr-applicability of, say, 'bad'): in this case a dispute may arise concerning the 'appropriateness' of one standard as against the other. An argument of this sort is itself evaluative, in that vpr-application of 'right' and 'wrong' to S or T implies ev-application of some standard to those standards. (145) This is not to claim either that nev-applied standards are never contrasted in this way or that a conflict of ev-applied standards may not be dismissed simply as a case of 'different standards', for it is indeed true both that evaluative arguments sometimes arise over the merits of conflicting nev-applied standards and that we sometimes fail to argue over conjointly ev-applicable standards which entail antithetical vpr-applicabilities (saying, e.g. 'Chacun à son goût'). On the other hand, it could be suggested that disputes over ev-applied standards are in principle, rationally irresolvable, whereas disputes over nev-applied standards - even evaluative disputes - must be rationally resolvable. I do not enquire into the merits of this suggestion here.

C. It is perhaps tempting to think that legal rules are involved in deciding cases and in issuing verdicts similarly as standards are involved in making evaluations. Grant suggests that verdicts and appraisals "usually consist in the application of accepted rules, principles and criteria to a particular case."⁽¹⁴⁶⁾ And O'Connor claims that many legal disputes "are carried on and settled by methods of reasoning which are typical of disputes in valuational matters," in that "here too we have the task of judging particular cases by reference to standards, in this case, statutes, legal rules and precedents."⁽¹⁴⁷⁾ On the other hand, it seems to me that legal rules and ev-applied standards differ in several respects. Whereas interpretation of law L is generally involved in deciding upon application of L, ev-application of standard S does not, in this way, require interpretation of S. Furthermore, what is at stake in ev-applying S to x is the vpr-applicability of some expression to x, whereas coming to a legal decision consists in declaring that a case sub judice falls under, or does not fall under a certain law: the jury in a criminal-law trial does not weigh evidence similarly as the judge in a flower show assesses criterial characteristics of a pot/

pot of begonias, and the jury-foreman's "Guilty" plays a correspondingly different role from that played by the flower-judge's "Excellent". ("First Prize" uttered by the flower-judge plays a role more like the jury-chairman's "Guilty": both are verdictive performatory utterances.) What follows from the foreman's utterance of "Guilty" is that the law in question is applied to the case (defendant), for only when it has been decided that law L is valid for (applies to) defendant x - i.e. only after performance of the verdictive activity - can L (properly) be applied to x; whereas a flower judge applies standard S to begonias y in appraising them (i.e. he ev-applies S to y), for he knows already that S is valid for (applies to) y and hence that S can (properly) be applied to y.

It will be objected against this view - viz. that law L is applied to case C only when L has been found to apply to C and not in deciding whether L applies to C - that it constitutes a perversion of the ordinary usage of 'apply' in legal contexts. If Hart's use of 'apply' in the following passage exemplifies this usage then the objection must be granted.

Most of the difficulty in applying legal rules to concrete cases arises where (a) there is no difficulty in citing clear or standard cases to which the rule undisputably applies, but (b) in a given case a difficulty is precipitated because some feature present or absent in the standard case is absent or present in this case. (148)

Hart apparently thinks of L as itself applicatively involved in the activity of deciding whether L applies to C. But if L were involved in this activity similarly as a standard is involved in the activity of evaluating in accordance with it, then we might expect to be able to say: standard S is applicatively involved in deciding whether S applies to x. On the other hand, suppose we think of L' as applicatively involved in the activity of deciding whether "Guilty of delict D" is applicable to C, where L' is of the form:

(L') "Guilty of D" is applicable to C on condition that C satisfies criteria A, B, C.

Since application of "Guilty of D" to C by the appropriate judge or jury-foreman is tantamount to a declaration that C is duly found to have committed D, we can reformulate L' as:

(L'') C is (duly) found to have committed D on condition that C is (duly) found to satisfy criteria A, B, C.

And since law L₀ of the (over-simplified) form:

(L) Sanction S is to be executed on x on condition that x is (duly) found to have committed delict D,

can be said to apply to C when C is (duly) found to have committed D, it follows that whether L applies to C is decided by applying L'' to C, where (the outcome of) this decision is expressed in "Guilty"

"Guilty (not guilty) of D". But law L itself is not applied to C in determining whether it applies to C - unless L is of the form L''',

(L''') Sanction S is to be executed on x on condition that x is (duly) found to have committed D, where commission of D is defined by criteria A, B, C;

though even here I should want to say that L''' is applied to C in a primary sense by the judge when he directs sanction S to be executed on C if C has been found guilty of D.

In the preceding analysis I have followed Kelsen's conception of legal rules as norms obligating execution of a sanction on condition of commission of a delict. Kelsen argues that legal rules immediately regulate verdict-issuing and sentence-passing activities of legal organs and only secondarily regulate the behaviour of citizens. He accordingly distinguishes between 'secondary' and 'primary' legal norms; the former of which prescribe "that a certain individual 'ought' to observe certain conduct", and the latter of which stipulate "that another individual ought to execute a sanction in case the first norm is violated." (149) Thus, corresponding to the primary norm L is the secondary norm: 'Doing D is forbidden' (i.e. 'D ought not be done'). Kelsen further distinguishes between 'applying' and 'obeying'.

'obeying' a legal rule: only the 'law-applying organ' for which a primary norm is 'valid' is said to 'apply' this norm by issuing a directive to the effect that a sanction is to be executed on a subject who has 'disobeyed' a secondary norm. It follows that the primary norm is 'valid' for citizens only indirectly and that, in consequence, they can only indirectly be said to 'obey' or 'disobey' it.⁽¹⁵⁰⁾ This strikes me as a neat - probably much too neat - way of explicating the relation of (criminal) law to citizens for whom it is 'valid'. L.J. Cohen criticises Kelsen's theory on grounds that it involves 'linguistic reconstruction' and "surrendering any attempt to stay close to the normal range of uses in law of key words like 'rule', 'create', and 'apply'."⁽¹⁵¹⁾ I do not know whether 'rule' and 'apply' have 'a normal range of uses in law', but it does not seem to me that Kelsen's usage of these terms is exceptionally odd, at least in the context mentioned above.^(151a)

An example of confusion in usage of 'apply' with respect to legal rules occurs in Baier's discussion of similarities between laws and customs. Claiming that one of the dimensions in which types of rules differ is 'the way they are applied', he writes:

There are no important differences in the way laws and customs are applied. It is part of the content of a law or custom to specify the groups of persons to whom it is meant to apply. Military draft regulations,... the custom that men should take off their hats in lifts... from their very nature apply to certain groups of persons only.⁽¹⁵²⁾

Baier's/

Baier's point that customs and laws both 'apply to' specifiable groups of persons is, of course, unexceptionable if understood merely as a claim that both customs and laws are 'relevant' for a range of persons mentioned in their formulations. But the sense of 'relevance' in the case of laws is quite different from its sense in the case of customs. Laws are applied to those for whom they are 'relevant', but customs are not 'applied' in this, or any other, sense to persons expected to follow them. Moreover, while laws are 'meant' to apply to certain persons, it is not clear to me that customs are 'meant' to apply; nor for that matter, am I clear that customs can be said to 'apply' to persons expected to follow them. There is some resemblance between 'following' customs and 'obeying' laws, but I am not sure whether this similarity or the 'relevance' similarity mentioned above, or indeed both, is what Baier has in mind here. At all events neither has anything to do with the 'application' of customs and legal rules - in either a transitive or an intransitive sense of 'apply'.

One final point, concerning constitutive legal rules. Without a framework of legal institutions there would be no sanction-stipulating legal rules of the form schematised above (I), since specification of sanction to be executed (i.e. sentence-passing) on condition of delict-commission is a legal activity only/

only to the extent that it is carried out by an authorised organ, i.e. within a legal system. Hart talks of "constitutional rules providing criteria valid for the system for the identification of rules of the system",⁽¹⁵³⁾ such rules are constitutive of legal institutions, since identification of precepts as rules of some legal system depends on the existence of an institutional framework within which they are enacted, applied, rescinded, &c. Note, incidentally, the similarity of Hart's characterisation of constitutional rules to my formulation of the notion of C-rules in terms of describability (identifiability) of behaviour-instances in terms of practices. On the other hand, in so far as constitutional rules delimit procedures in accordance with which legal organs carry out their functions, they serve also as R-rules.

[4] Sentences in group (III) (p. 99, above) exemplify contexts which have to do with applicative involvement of theoretical, mathematical and logical systems, and components of these systems (e.g. laws of nature, hypotheses, mathematical theorems, logical theses, theoretical and abstract concepts, &c.). In this section I deal only with application of theories and laws of/

of nature to events and phenomena, and with application of mathematics to objects. In Chapter V, I touch upon applicative involvement of logical theses.

A. Predicting and explaining phenomena and events are foremost among activities in which (physical) theories are applicatively involved. If we think of 'predicting event E in terms of theory T' and 'explaining event E in terms of theory T' as analogous to 'evaluating \underline{x} in terms of standard S', then we can introduce 'prd-applying T to E' and 'exp-applying T to E' as analogous to 'ev-applying S to \underline{x} '. According to the 'hypothetical-deductive' (hereafter 'H-D') view of explanation and prediction, these two modes of theory-application are distinguished 'pragmatically' and not 'logically': in Popper's words, "The use of a theory for predicting some specific event is just another aspect of its use for explaining such an event";⁽¹⁵⁴⁾ and Gardiner's, "The 'logical structure' of explanations of this kind is the same as that of predictions," where by 'of this kind' he means an explanation that consists in "(1) stating a universal law, or set of laws, (2) stating the existence of a set of initial conditions G_1, \dots, G_n , so that from these two statements a third statement describing the event in question follows."⁽¹⁵⁵⁾ Here, in terms of prd=/

prd- and exp-application, is a schematic outline of prediction and explanation as seen by advocates of the H-D view.

- (α) To prd-apply T to E is to deduce E', a statement predictive of E, from T and S', where S' is a set of statements of initial conditions S.
- (β) To exp-apply T to E is to determine initial conditions S, stated in S', such that E', a statement descriptive of E, is deducible from T and S'; in such a case we are said to explain E in terms of T by adducing S as 'determining conditions' of E.

It can be seen that in both (α) and (β), deduction, from T and S', of a statement descriptive (predictive) of E is of primary logical importance. H-D theorists sometimes talk of another mode of theory-application. Sentence (a) in group (III), for example, is "The lowest-level hypothesis IIIa. is tested by applying it to a particular case"; Popper follows this usage of 'apply' in characterising theory-testing:

A theory is tested not merely by applying it,...
but by applying it to very special cases - cases
for which it yields results different from what we
should have expected without the theory, or in the
light of other theories. (156)

It is evident that in both prd- and exp-applying T to E, T is applied, in this third sense, to S. Suppose, for instance, that T_1 is of the form $(\underline{x})(\underline{F}\underline{x} \supset \underline{G}\underline{x})$, and S'_1 , which states S₁, is a statement 'Fa'; then to apply T_1 to S'_1 is (i) to infer, by universal instantiation on T_1 , the statement 'Fa \supset Ga'; and/

and (ii) to infer, by modus ponens on ' $Ta \supset Ga$ ' and S_1^i , the statement ' Ga '. (157)

Proponents of what I shall call a 'quasi-instrumentalist' view of theory-application hold, as against the H-D view, that (in the words of Toulmin):

Laws of nature do not function as premises from which deductions to observational matters are made, but as rules of inferences in accordance with which empirical conclusions may be drawn from empirical premises. (158)

But it should not be thought that the quasi-instrumentalist differs from the deductivist merely in holding that pred-application of T to E consists in inferring E' from S' in accordance with T, and that exp-application of T to E consists in determining S such that E' is inferable from S' in accordance with T. On the quasi-instrumentalist view, a theory provides not only 'inferring techniques' in accordance with which events are predicted, but also 'models' and 'methods of representation' in terms of which events to which the theory is exp-applied are rendered intelligible - are explained. As Dray says:

Some theories, we must admit, may be just inferring techniques, since they may lack a model. But if there are any such, perhaps we should think twice about calling them explanatory theories; at most they 'explain' in the technical sense. (159)

For theories qua predictive devices, 'inferring techniques' are of primary importance, but for theories qua devices for making data/

data intelligible, what is important is that theories provide "an intelligible, systematic, conceptual pattern for the observed data," whose value "lies in its capacity to unite phenomena which, without the theory, are either surprising, anomalous, or wholly unnoticed." (160) This pattern may be provided in a 'picturable' model (as e.g. in the kinetic theory of gases and in geometrical optics), or, where (as in elementary particle physics) there is 'unpicturability-in-principle', it may be provided by mathematical techniques and symbolism. Hanson remarks, apropos of the latter sort of 'pattern':

Mathematical techniques more subtle and powerful than the geometry of Kepler, Galileo, Beeckman, Descartes and Newton are vital to to-day's physical thinking. Only these techniques can organise into a system of explanation the chaotically diverse properties which fundamental particles must have if observed phenomena are to be explained. (161)

The quasi-instrumentalist holds, then, that to *pré*-apply T to E is to infer E' from S' in accordance with T; but he denies that *exp*-application of T to E is merely *pré*-application 'upside-down', for he claims that T's explanatory power is, to a certain degree, logically independent of its usefulness as a predictive device.

In view of this fact, it is important to distinguish the quasi-instrumentalist position from the view labelled 'instrumentalism' by/

by Popper. He accuses proponents of this view of caring about nothing but "(a) mastery of the mathematical formalism, i.e. of the instrument, and (b) its applications." (162)

Instrumentalism thus subverts the Galilean doctrine that theories are "not only instruments but also (and mainly) descriptions of the world," (163) for it regards 'laws' as "nothing but computation rules (or inference rules), fundamentally of the same character as the computation rules of the so-called 'applied' sciences." (164)

While the quasi-instrumentalist may, as we have seen, think of theories as inference rules rather than as true-or-false premisses from which empirical statements are deducible, he cannot be accused of regarding theories solely as predictive devices; for he insists that theories should help to render observable data intelligible, as well as subserve predictive inference. Popper, on the other hand, would (perhaps) argue that this is not enough, that theories must be regarded as true-or-false descriptions of the world, and not just as loci of conceptual patterns whereby data are rendered intelligible.

He states, for example:

The scientist aims at finding a true theory or description of the world (and especially of its regularities or 'laws'), which shall also be an explanation of the observable facts. (This means that a description of these facts must be deducible from the theory in conjunction with certain statements, the so-called 'initial conditions'.) (165)

An/

An argument between Popper and a quasi-instrumentalist concerning the relative importance of the descriptive, as against explanatory, function of a theory, might degenerate into a quibble over the meanings of 'describe' and 'explain'. For surely, in providing a conceptual pattern for the ordering of phenomena, a theory can be said to play a descriptive role; and to carry out what Popper regards as the scientist's aim is to construct theories which help explain - in the quasi-instrumentalist's sense - phenomena. Nonetheless, if one begins by thinking of theory-construction in terms of formulating 'conceptual gestalts', then he is unlikely to view theories as primarily descriptive in function: emphasis on 'models' and on the orderly colligation of phenomena is apt to prove an insurmountable obstacle to regarding theories in this way. This is not, of course, to say that quasi-instrumentalists deny that theories have a descriptive function: as Toulmin says, "the laws themselves do not do anything: it is we who do things with them." Physicists use Newton's Laws "to describe, say, the way a shell moves" - the laws, however, "do not set out by themselves to tell us anything about the actual motions of particular bodies, but rather provide a form of description to use in accounting for these motions." (166) It is likely Popper would argue that, on the/

the contrary, Newton's Laws do in themselves describe certain of the world's 'regularities or laws', (167)

Philosophers of science sometimes talk of the 'range' or 'scope' of application (applicability) of theories and laws. Toulmin makes much of a distinction between statements of scope and statements of the theory or law itself, claiming that this distinction is characteristic of all physical theories and laws. It is worth noticing various ways in which he discusses the notion of 'scope'. Statements of scope are said to be of the form "X's law has been found to hold, or not to hold, for such-and-such systems under such-and-such circumstances." (168) Laws of nature "are the sorts of statements about which it is appropriate to ask, not 'Is it true or not?' but rather 'To what systems can this be applied?' or 'Under what circumstances does this hold?'" (169) Again, Toulmin notes a "division of labour in physics, between laws themselves and statements about the ways in which, and the circumstances in which laws are to be applied." (170) To say that a theory has a limited scope is to say that "only a limited range of phenomena can be explained using that theory." (171) We should distinguish between talking about a theory, e.g. "recognising a situation as one in which a particular theory can be employed," and talking in terms of a theory, e.g. "employing the theory in that situation on the assumption/

assumption that it has been correctly identified." (172)

Toulmin repeats this distinction, contrasting "identifying a system as one to which [a theory] applies" with "applying it to explain or foretell the phenomena occurring in such-and-such a situation." (173) Finally, the physicist's predilection for making "a limited number of observations covering a wide range of circumstances" is said by Toulmin to manifest his aim "to discover the scope of the theory, not its degree of truth or the conditions on which it can be accepted as true." (174)

Several ambiguities are evident in these statements: a statement of scope is said to mention both 'systems' or 'phenomena' for which, as well as 'circumstances' or 'situations' in which, a theory or law either 'holds' or 'can be employed'; statements of scope are also said to assert 'the ways' and 'circumstances' in which laws 'are to be applied'. Note too that such a statement asserts both that a law has been found to hold, and that a law does hold: the first is, I should have thought, a different statement from the second, since it merely reports an experimental discovery whereas the second makes a claim apropos of a proposed application of the law. Against this, Toulmin would argue that the fact that a law L has been found to hold for systems of type S under conditions of type C is prima facie a good reason for presuming that it does hold (can be applied to) system S' under conditions C', where/

where S' and C' resemble S and C in 'physically significant' respects. The astrophysicist, for instance, presumes the same laws to be exp-applicable to motion of the parts of a double-star as have been found to exp-apply to such phenomena as the motion of falling apples and of the satellites of Jupiter: he makes this presumption on the strength of what Toulmin describes as 'structural resemblances' between systems within which, and conditions under which, phenomena are observed to take place. This is to follow the methodological precept: "Unless there is some reason to suppose that a novel phenomenon cannot be explained in terms of the theory which it is natural to turn to first, there is every reason to turn first to that theory."⁽¹⁷⁵⁾ As Toulmin justly remarks, this is not a very dangerous precept.

Popper, as might be expected, attacks such a conception of 'scope'. He argues that since, "for instrumental purposes of practical application, a theory may continue to be used even after its refutation, within the limits of its applicability," it follows that "the instrumentalist interpretation will therefore be unable to account for real tests, which are attempted refutations, and will not get beyond the assertion that different theories have different ranges of application."⁽¹⁷⁶⁾ Hence instrumentalism/

instrumentalism cannot account for scientific progress, and
"may well be responsible for the recent stagnation in
(176a)
theoretical physics." Quoting Heisenberg as having said:

We do not say any longer: Newton's mechanics is false...
Rather we now use the following formulation: 'Classical
mechanics... is everywhere exactly 'right' where its
concepts can be applied';

Popper comments:

Since 'right' here means 'applicable', this assertion
amounts precisely to saying 'Classical mechanics is
applicable where its concepts can be applied' - which
is not saying much. (177)

Unfortunately, however, Popper misquotes Heisenberg, and it
seems to me that what the latter actually says cannot be so easily
turned into a truism:

Classical mechanics... is everywhere a strictly 'correct'
description of nature, where its concepts can be applied.

Heisenberg goes on to explain that the clause "where its
concepts can be applied" is meant only to indicate "that we
consider the range of application of Newtonian theory to be
limited." (178) This is surely unexceptionable, and not so
trivial as Popper's comment suggests. Indeed, it is not, on
Popper's own showing, an 'instrumentalist' remark at all, for
Heisenberg is not saying that Newtonian physics may be used
'for instrumental purposes of practical application' within
limits of its applicability: he is rather asserting - what I
should/

should think Popper would wholeheartedly endorse - that within limits of its applicability (i.e. applicability of its concepts), Newtonian physics gives a true description of the world.

B. Philosophers frequently address themselves to problems which fall under the heading 'application/applicability of mathematics to the world (reality, facts, experience, &c.)'. Here, to supplement sentences of group (III), is an assortment of remarks concerning these problems.

- (a) This transcendental principle of the mathematics of appearances.... alone can make pure mathematics, in its complete precision, applicable to objects of experience. (Kent)
- (b) The laws of number, therefore, are not really applicable to external things; they are not laws of nature. They are, however, applicable to judgements holding good of things in the external world: they are laws of the laws of nature. (Frego)
- (c) Whether a geometry can be applied to the actual physical world or not, is an empirical question which falls outside the scope of the geometry itself. (Ayer)
- (d) It is a mere matter of fact that a certain system of arithmetic applies to the world in the way it does apply: another system of arithmetic could have been applied to the world without the world's being different in any other way: but it would have applied differently. (Britton)

(e)/

- (e) Thus the problem of application of euclidean geometry to physical space turns out to be illusory, and it must be replaced by the wider problem of application to the physical world of a comprehensive formalism which covers both geometry and mechanics - the formalism of the theory of relativity, for example. (Kneebone) (179)

It may also be useful to give examples of statements which mention mathematical devices (i.e. systems and components of systems) as instrumentally or quasi-instrumentally involved in activities in which they might be said to be applicatively involved. Bridgman, for example, talks of "the use of mathematics in describing nature"; (180) Popper observes "the calculus of natural numbers is used in order to count billiard balls," whereas "the calculus of real numbers provides a framework for measurements of continuous magnitudes such as geometrical distances or velocities"; (181) Ayer says "we can use a geometry to reason about physical space" since, after interpreting its axioms, "we can proceed to apply the theorems to the objects which satisfy the axioms"; (182) and Carnap describes a mathematical theorem as "an instrument facilitating operations with factual statements, namely, the deduction of a factual conclusion from factual premisses." (183) Needless to say, many more examples of such statements could be adduced.

In the space (and time) now at my disposal I am unable to deal adequately with this vast topic. Two general questions are of special significance in this context. One concerns the relation of interpretation of mathematical systems to their application; this is particularly important with respect to geometrical calculi (rather: calculi interpretable as physical geometries). The other, a somewhat less technical and more philosophical issue, concerns the Kantian conundrum: Why is it possible to apply pure mathematics to the world (experience, reality, &c.)?!. Many intriguing remarks have been made, by both mathematicians and philosophers, about this second question - for example, Collingwood's remark apropos of the 'Renaissance conception of nature':

Thus, the possibility of an applied mathematics is an expression, in terms of natural science, of the Christian belief that nature is the creation of an omnipotent God. (184)

Rather than pursue these interesting and important questions, however, I comment briefly on the applicative involvement of arithmetical propositions.

According to Frege, the proposition 'if we pour 2 unit volumes of a liquid into 5 unit volumes of a liquid we shall have

7 units volumes of liquid' is "an application" of the proposition ' $5 + 2 = 7$ '.⁽¹⁸⁵⁾ Much the same point is made by Popper, who says that ' $2 + 2 = 4$ ' can be 'applied' in two senses to objects such as apples.⁽¹⁸⁶⁾ In the first sense, "the application is not real but only apparent" since "we do not describe here reality, but only assert that one way of describing reality is equivalent to another way." This mode of 'application' is exemplified in the statement '2 apples + 2 apples = 4 apples' (A). In the second sense, on the other hand, "the statement ' $2 + 2 = 4$ ' helps us to calculate, i.e., to describe certain physical facts, and the symbol '+' stands for a physical manipulation - for physically adding certain things to other things." This second mode of application is exemplified in the statement 'If somebody has put two apples in a certain basket, and then again two, and has not taken any apples out of the basket, there will be four in it' (B). Keene makes a similar distinction between the two statements (I) 'Two shillings plus two shillings equals four shillings', and (II) 'If you put two shillings in a purse containing two shillings then you will have four shillings in it': he calls (I) "an application of ' $2 + 2 = 4$ ' in a very weak sense," and suggests that "until the words 'plus' and 'equals' in (I) are given a physical/

physical meaning or replacement... the statement (I) is, so to speak, a mongrel-empirical, not a purely empirical statement."⁽¹⁸⁷⁾ Popper's reason for calling (B) an 'application' in a 'real' sense of ' $2 + 2 = 4$ ' is similar to Keene's reason for according this status to (II): Popper claims that ' $2 + 2 = 4$ ' as interpreted in (B), "becomes a physical theory, rather than a logical one." We find Hamlyn arguing along the same lines as Frege, Popper and Keene: "How does ' $2 + 2 = 4$ ' have an application? Surely by enabling one to derive such statements as 'If you put two apples with another two apples you will get four apples'."⁽¹⁸⁸⁾ What does Hamlyn mean by 'derive' here? He later writes:

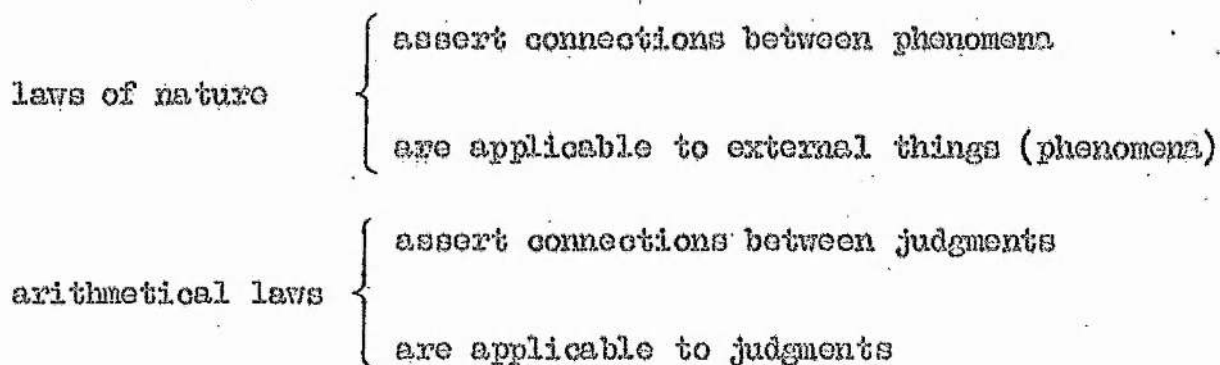
There is an essential connection between " $2 + 2 = 4$ " and its application. It is because the formula " $2 + 2 = 4$ " must hold that we can say that the statement which comprises its application must be true.⁽¹⁸⁹⁾

Since he also talks of 'contingencies in which the formula does not apply', we might reformulate this statement: If ' $2 + 2 = 4$ ' applies to (holds for) X's then the statement 'If two X's are put with another two X's you will get four X's' must be true. The formula does not apply to drops, but it does apply to apples: therefore it can be applied to apples but not to drops. To apply ' $2 + 2 = 4$ ' to X's may be either (1) to assert that there will be four X's if two X's are put with another two X's, or (2) to infer that there will be four X's on grounds that two X's are (have been) put/

put up with another two X's. Hence, to say that the formula can be applied to apples is to say that (1) and (2) are permitted in case X's are apples; and to deny its applicability to drops is to deny that (1) or (2) is permitted in case X's are drops. The relation between (1) and (2) is that of inference-warrant to inference: to justify an inference that there will be four X's, &c., one may appeal to the 'fact'/'rule' that if two X's are put with another two X's, &c.

Frege argues that 'the laws of number' (e.g. ' $2 + 2 = 4$ ' interpreted qua rule) "assert not connections between phenomena, but connections between judgments," though since "among judgments are included the laws of nature," and since "every proposition of arithmetic [is] a law of logic, albeit a derivative one," he concludes that "to apply arithmetic in the physical sciences is to bring logic to bear on observed facts." (189a)

Frege's contrast between arithmetical laws and laws of nature is represented in this diagram:



Now, /

Now, in line with this distinction, we might expect Frege to argue against the analysis of the preceding paragraph: ' $2 + 2 = 4$ ' is not 'applicable' to apples and 'inapplicable' to drops, for to apply ' $2 + 2 = 4$ ' to X's is really to apply this law to (1) judgments that there will be four X's if two X's are put with another two X's, or to (2) judgments (a) that there will be four X's and (b) that two X's are (have been) put with another two X's, where judgment (a) is made on the basis of (deduced from) judgment (b). And, Frege might go on to say, ' $2 + 2 = 4$ ' asserts the connection between judgments (a) and (b) in virtue of which (a) is (can be) judged on the basis of judgment (b). It follows that in applying ' $2 + 2 = 4$ ' to (1) or to (2) one is 'bringing logic to bear' on 'observed facts', for the judgments which comprise (1) and (2) can be said to 'hold good of things in the external world', namely X's. But, of course, a proposition that expresses the content of (1) is true only if X's are e.g. apples, untrue if X's are e.g. drops; similarly, deduction of judgment (a) from judgment (b) is valid only if X's are e.g. apples, invalid if X's are e.g. drops.

In the first of the two following essays I supplement my discussion of the application of arithmetical formulae and laws by considering Wittgenstein's views on this and other closely related topics. In the second essay I deal with some problems concerning the application of rules of inference.

[1] Arguing against the formalists in Grundgesetze, Frege claims that "an arithmetic with no thought as its content will also be without possibility of application." He goes on to argue:

Why can no application be made of a configuration of chess pieces? Obviously, because it expresses no thought. If it did so and every chess move conforming to the rules corresponded to a transition from one thought to another, applications of chess would also be conceivable. Why can arithmetical equations be applied? Only because they express thoughts. How could we possibly apply an equation which expressed nothing and was nothing more than a group of figures, to be transformed into another group of figures in accordance with certain rules? Now, it is applicability alone which elevates arithmetic from a game to the rank of a science. So applicability necessarily belongs to it. Is it good, then, to exclude from arithmetic what it needs in order to be a science? (190)

Frege admits that the formalists, Thomae for example, contrast "the arbitrary rules of chess with the rules of arithmetic"; but he complains that they do this only in terms of the applicability of arithmetic, whereas within the domain of formal arithmetic/

arithmetic "its rules appear as arbitrary as those of chess." (191)
The formalist, in effect, shifts this problem - the apparent non-arbitrariness of arithmetic - "to the shoulders of his colleagues, the geometers, the physicists and the astronomers," (191a) who rightly decline to attack it since, in view of the manifold applications of arithmetical formulae, "it is likely that the problem of the usefulness of arithmetic is to be solved independently of those sciences to which it is to be applied." So it is that, "in order to bridge the gulf between arithmetical formulae and their applications, it is necessary that formulae express a sense and that the rules be grounded in the reference of the signs."

It is noteworthy that Frege frames his quarrel with the formalists in terms of the meaning of mathematical formulae. What distinguishes mathematics from a mere game, he claims, is its applicability, and the only way to account for this difference is to remark the difference in significance between formulae and calculations as against chess configurations and chess moves: it can be seen that Frege locates this difference in the fact that the former 'express thoughts' whereas the latter do not. Frege contrasts 'meaningful' with 'formal' arithmetic, (192) the latter, since its numerical signs have no reference (Bedeutung), consists/

consists of truth-valueless and senseless formulae, for only signs which "have been given a reference can be used in sentences expressing true thoughts,"(193) Compare Frege's interest in the meaningfulness of mathematical propositions with Wittgenstein's dictum: "What mathematical propositions do stand in need of is a clarification of their grammar."(194) Significantly, this comes as an answer to the question "What does mathematics need a foundation for?". That mathematics does need a foundation - a foundation in logic - is, of course, a starting point of Frege's programme: Wittgenstein's answer here is, in effect, a repudiation of the programme. Typical of his arguments against it is this:

The reduction of arithmetic to symbolic logic is supposed to show the point of application of arithmetic, as it were the attachment by means of which it is plugged into its application.... But the attachment.. is on the one hand too narrow, on the other hand too wide; too general and too special. The calculation takes care of its own application.(195)

Which may be understood to be directed against such remarks as Frege's:

Arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one. To apply arithmetic in the physical sciences is to bring logic to bear on observed facts; calculation becomes deduction.(196)

Preliminary/

Preliminary to considering the views of Wittgenstein himself, I want to give a brief exposition of Waismann's objections to the above-quoted argument of Frege. Waismann acknowledges indebtedness to Wittgenstein for at least one of his objections, and it seems to me that the latter's influence is evident in others as well.

Waismann applauds Frege's attention to the question of application of arithmetical formulae, but gives his plaudits a characteristically Wittgensteinian twist by claiming "if one disengages himself wholly from the application, and if one breaks all ties with verbal language, then surely one is merely playing a game."⁽¹⁹⁷⁾ He objects, however, to Frege's insistence that behind the applicability of mathematics lies the fact that mathematical formulae express thoughts. When asked for the meaning of a formula we give examples of its application and render the formula verbally: if it has neither application nor verbal expression, then Waismann says we should seek to incorporate it in some 'wider syntactic context!'.⁽¹⁹⁸⁾ This means, I think, that we should give it a place in some 'language-game' either by extending an existing 'game' or by creating a new one (though Waismann would perhaps claim that extension just is creation in this case). As to instances of the first type - in which the meaning of a formula is explained by giving examples of its application and by/

by rendering it verbally - Weismann asks whether a child taught the meaning in this way doesn't yet grasp the 'proper sense' of the formula and is able to make only 'mechanical' use of it. It is somewhat surprising, as Dummett notes in a review of Weismann's book, to find Frege accused - by implication if not explicitly - of 'psychologism'. (199) Weismann uses the phrases and 'ein Vorgang des Verstehens' 'ein geistiger Vorgang' / in discussing Frege's 'expression of thoughts', and he writes:

But when someone asks me what " $1 + 1 = 2$ " means, then I won't answer with a description of my mental condition, but with an explanation of the sign. (200)

To interpret Frege's phrase 'expression of thoughts' as symptomatic of psychologism is, I think, seriously to misconstrue Frege.

In dealing with the passage quoted above, wherein Frege contrasts arithmetic with chess and concludes that the latter lacks application because it expresses no thoughts, Weismann argues that, on the contrary, application alone accounts for the significance of arithmetical equations as against configurations of chess-men. Thus we should not say: 'Since a chess configuration expresses no thought, therefore it lacks application'; but rather, 'Since we have provided it with no application, therefore a chess configuration expresses no thought.' (201) If arithmetical equations did/

did not subserve inference from one (non-arithmetical) proposition to another, then arithmetic would be a mere game. (202)

Chess configurations might have application, e.g. a general might plan a battle on a chess board, and in such a case they are not mere positions in a game, but can be said to have sense. This shows, Waismann claims, that application alone accounts for the significance of arithmetic as against chess. (The idea that 'mathematical propositions subserve inference' is pervasive throughout Wittgenstein's Remarks, and is, I believe, acknowledged by Waismann to be due to Wittgenstein.)

The Remarks on the Foundations of Mathematics contains, in bits and pieces, Wittgenstein's philosophy of mathematics (more precisely, perhaps, it contains Wittgenstein's attempts to clarify his dictum that what mathematical propositions need is a clarification of their grammar). My primary aim, in what follows, is to discuss the rôle Wittgenstein assigns to 'application' as a determinant of the meaning of mathematical propositions: in the course of my discussion, however, I consider other central themes of Remarks, in particular the relation of 'proof' to the 'grammar' of mathematical propositions.

Mathematics, according to Remarks, is "a motley of techniques of proof," and upon this fact "is based its manifold applicability and/

and its importance": for it "teaches us to operate with concepts in a new way," it "forms concepts" and "forms a network of norms": where "'to give a new concept' can only mean to introduce a new employment of a concept, a new practice," and "concepts correspond to a particular way of dealing with situations." (203)

Contrasting mathematics and games, Wittgenstein says: "it is the use outside mathematics, and so the meaning of the signs, that makes the sign-game into mathematics"; this connects with his remark "it is essential to mathematics that its signs are also employed in mufti." (204) That is to say, mathematical formulae subserve non-mathematical inference, e.g. in making predictions, building bridges and measuring distances; they play this rôle because they are regarded as rules or standards, and their status as rules or standards derives from their having been proved. Hence the notion of proof is crucial in Wittgenstein's philosophy of mathematics: the 'must' which a mathematical proof introduces is said to correspond "to a track which I lay down in language," and "mathematical propositions are instruments taken up into the language once for all - and their proof shews where they stand." (205) (Several paragraphs later, however, the phrase 'instrument of language' is applied to proof and to calculations; (206) Wittgenstein's tendency to throw 'mathematical proposition', 'proof' and 'calculation' all into the same basket is perhaps a reflection of/

of his dictum: "In mathematics process and result are equivalent." (207)

Concerning the nature of mathematical proof, Wittgenstein says "proof must be a procedure plain to view," "what is proved by a mathematical proof is set up as an internal relation and withdrawn from doubt"; he warns against regarding the proof as "a procedure that compels you," suggesting that it is rather a procedure which "guides... your conception of a (particular) situation"; "the proof as it were guides our experience into definite channels," in that it "moulds our language" and "leads me to say: this must be like this." (208) Proof connects with 'application' in this way:

The proposition proved by means of the proof serves as a rule - and so as a paradigm. For we go by the rule. But does the proof only bring us to the point of going by this rule (accepting it), or does it also shew us how we are to go by it?

For the mathematical proposition is to shew us what it makes SENSE to say.

The proof constructs a proposition; but the point is how it constructs it. Sometimes, e.g., it first constructs a number and then comes the proposition that there is such a number. When we say that the construction must convince us of the proposition, that means that it must lead us to apply this proposition in such-and-such a way. That it must determine us to accept this as sense, that not. (209)

It can perhaps be gathered from the last two sentences of this paragraph that application of the proposition in such-and-such

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a way is manifested in acceptance of this as sense, that as nonsense. What determines how we apply the proposition (what we accept as sense, what not) is not that the proposition is proved, but rather the way in which, or how, it is proved.

Proof, one might say, does not merely shew that it is like this, but: how it is like this.

The proof (the proof-picture) shows us the result of a procedure (the construction); and we are convinced that a procedure regulated in this way always leads to this picture. (210)

What convinces us (i.e. what makes the proof-picture a proof-picture for us) lies open to view: "We see in the proof the reason for saying that this must be the result." (211) It is not that the proof-picture is like this which leads us to accept the picture as a proof, but rather how the picture yields the result. When, for example, we regard Figure (a) as proof that a rectangle can be made of two parallelograms and two triangles, we do not see it merely as a picture of two parallelograms and two triangles arranged to form a rectangle: we rather see in it how a rectangle is to be (can be) constructed from such shapes. (212) That is to say, we grasp a technique or procedure whereby to construct a rectangle from triangles and parallelograms. Our conviction that a rectangle can be so constructed derives from our seeing

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a reproducible procedure in the proof-picture; in seeing how to make the construction in (a), we see (a) as a proof-picture of the possibility of (making) such a construction for any rectangle (where 'any' signifies that this possibility is a mark of the concept of rectangle).⁽²¹³⁾

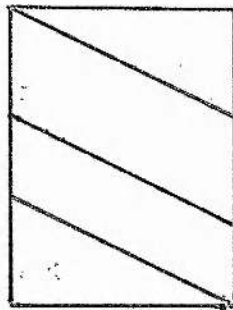


Figure (a)

The 'inexorability' of this proof-picture - that it leads us to say 'Q.E.D.' and regard it as established that a rectangle can be constructed of two parallelograms and two triangles - derives from our giving it proof-status, and this is manifested in what we do with the result.

Here we have something that looks inexorable -. And yet it can be 'inexorable' only in its consequences! For otherwise it is nothing but a picture.⁽²¹⁴⁾

Wittgenstein urges in another context that, as regards the difference between certainty that "I cannot know what is going on in him" and certainty deriving from mathematical demonstration, "the kind of certainty is the kind of language game."⁽²¹⁵⁾

(Another/

(Another instance of this line of thought is the question "How is 'the certainty that this is the case' manifested in human action?")⁽²¹⁶⁾ The point here - and a leading thesis of Remarks - is that to understand the certainty with which we regard mathematical propositions one should look at the application of these propositions - the language-games and activities, the forms of life in which they play a part and which they actually shape - rather than at psychological concomitants of accepting a proposition as proved. Certainty appertaining to mathematical propositions is distinguished in point of these factors from that appertaining to such statements as "I cannot know what is going on in him", and not in point of psychological factors.⁽²¹⁷⁾ But in both cases conviction is manifested in what we regard as sense, and what as nonsense, in what we regard as possible, and what as impossible: this determination of our concepts comes out in the application we make of the propositions.

Here once again we come back to the expression "the proof convinces us". And what interests us about conviction here is neither its expression by voice or gesture, nor yet the feeling of satisfaction or anything of that kind; but its ratification in the employment of what is proved.⁽²¹⁸⁾

In Remarks IV-25 Wittgenstein considers what it is to understand the proposition 'proved' by a non-constructive existence proof, e.g. "a proof [that] convinces you that there is/

is a root of an equation (without giving you any idea where)."

While, he says, you may "be convinced that the application of the proved proposition will turn up," nonetheless "you do not understand the proposition so long as you have not found the application." He repeats this in a slightly different way, saying that one understands a mathematical proposition formed "in a grammatically correct way" only "when one can apply it." But this answer is twice again reformulated: first, "when one has a clear picture of its application"; and secondly, "when one commands a clear view of its application." Wittgenstein is dissatisfied with the first reformulation because "it is not enough to connect a clear picture with it," and he considers the second reformulation to be "bad, for the matter is simply one of not imagining that the application is where it is not, of not being deceived by the verbal form of the proposition." By 'verbal form of the proposition' he means the sentence "there is..." which is the Wortausdruck of what the existence-proof purports to establish. Returning to this question in IV-46, Wittgenstein asks whether "I understand the proposition 'There is...' when I have no possibility of finding where it exists," and answers: "I understand it so far as I can explain it" (i.e. so far as I can explain it qua grammatically correct English sentence); but since/

since I cannot do with it (sc. with its proof) "what I can do with a constructive proof," and in so far as "what I can do with the proposition is the criterion of understanding it," it follows that "it is not clear in advance whether and to what extent I understand it." What can be done with (a proposition proved by) a constructive existence proof is to 'locate', through application of the proof-procedure, what it convinces us that there is. For example, suppose a proof 'establishes' that the pattern ' ϕ ' occurs in the expansion of π but does not show where (i.e. does not give a procedure for 'locating' it).⁽²¹⁹⁾ Not even God, Wittgenstein claims, could know whether this pattern occurs at, say, the n^{th} position of the expansion: even He cannot see in the rule of expansion alone that ' ϕ ' does (does not) occur at the n^{th} place, since "even God can determine something mathematical only by mathematics."⁽²²⁰⁾ This argument - against the 'picture' of occurrence or non-occurrence of a pattern at the n^{th} place of an expansion being already settled in the rule of expansion - connects with Wittgenstein's discussion of 'understanding the meaning of a word': as the 'mere rule of expansion' for π does not tell us whether or not ' ϕ ' is to be written at the n^{th} place, so the future employment of a word is not somehow 'present' in an 'act' of grasping its meaning. In Investigations Wittgenstein warns against inferring, from the premisses (1) we certainly understand

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a word and (2) its meaning lies in its employment, the conclusion (3) its "future development must in some way already be present in the act of grasping the employment and yet isn't present."⁽²²¹⁾ Similarly, he would warn against inferring from the fact that given a rule of expansion we can determine the nth place of the expansion, to the conclusion that this determination is somehow already present in the rule, as it were, so that to demonstrate the existence of some pattern at the nth place is to make a discovery of what is already settled in the rule. What we are given in the rule of expansion for π , and what we have in understanding the meaning of a word, is: a technique. Only by applying a calculating-technique do we establish that ' ϕ ' comes at the nth place of the expansion, and only by exercising language-techniques do we manifest our grasp of a word's meaning.

To return now to the problem of understanding a non-constructively 'proved' existential proposition, we recall that Wittgenstein sets out the criterion of understanding a mathematical proposition in three ways:

- (1) One understands P only when he 'can apply it' (wenn man ihn anwenden kann).
- (2) One understands P only when he 'has a clear view of its application' (wenn man ein klares Bild von seiner Anwendung hat).
- (3) One understands P only when he 'commands a clear view of its application' (wenn man eine klare Übersicht von seiner Anwendung hat).

We recall too that he says "what I can do with the proposition is the criterion of understanding it." But in so far as the proof of P determines what can be done with P, the question of understanding P is bound up with that of proving P: indeed, Wittgenstein seems to hold that the proof gives P its meaning. To illustrate the interconnection among 'understanding P', 'proof of P' and 'application of P', let us suppose an intuitionist mathematician to 'go through' two proofs of P: he claims not to understand P as 'proved' by the first, a non-constructive proof which involves an Axiom of Choice at some stage; but he claims to understand P as (constructively) proved by the second, which does not involve an Axiom of Choice. (222) The question is, in what sense is the 'application' of P made clear by the second proof, but not by the first? Or, alternatively, in what sense is one 'in a position to apply' P as established by the second proof in contrast to the first?

Wittgenstein's answer to this question would, I think, involve the difficult notions of 'perspicuity' and 'reproducibility' of proofs, and the view that mathematical propositions lay down 'conceptual connections' (Begriffsverknüpfungen) in virtue of their proofs. With considerable diffidence I try now to outline Wittgenstein's treatment of these ideas, thereby to throw light on 'application'.

'application' qua determinant of the meaning of mathematical propositions. Several passages from Remarks are pertinent:

(a) How far does the application of a mathematical proposition depend on what is allowed to count as a proof of it and what is not?

(b) For is the sense, the point, of a mathematical proposition really clear as soon as we can follow the proof?

When two proofs prove the same proposition it is possible... to imagine the proofs without the organism of applications which envelopes and connects the two of them...

(c) The proof is part of the surroundings of the proposition.

(d) And why is it mathematics? - Because it is a game with signs according to rules?
But isn't it evident that there are concepts formed here - even if we are not clear about their application?
But how is it possible to have a concept and not be clear about its application?(223)

Wittgenstein is suggesting, albeit more by innuendo and pointed question than by statement or argument, that the meaning of a mathematical proposition is essentially connected with its proof; for our understanding shows itself in the concepts or pictures which, in accepting the proposition as proved in this way, we adopt; and our adoption of these, in turn, is manifested in our granting the proposition 'the dignity of a rule', that is to say in our acting/

acting upon it. (224) The intuitionist claims to fail to understand P as 'proved' by the first proof, in spite of his being able to follow the 'proof' and despite the fact that he understands P as proved by the second proof; he makes this claim on grounds that the proof does not present a 'procedure plain to view': it lacks 'perspicuity', and hence 'reproducibility', since one cannot 'see in the proof the reason for saying that this must be the result' and hence "a doubt can make its appearance whether this is really the pattern of this proof... we are prepared to doubt the identity of the proof and so the derivation has lost its proving power." (225)

When I wrote "proof must be perspicuous" that meant: causality plays no part in the proof. Or again: a proof must be capable of being reproduced by mere copying. (226)

The 'proof' involving an Axiom of Choice lacks reproducibility because it lacks perspicuity: it is impossible to say of an alleged version of the 'proof' that it really proves P, for it cannot be guaranteed that a set postulationally selected in one alleged version plays the same rôle in it as that played by a set similarly selected in another. Working through such a 'proof' would be more like going through an experiment than like checking a calculation; and arriving at its outcome, P, would be correspondingly/

correspondingly similar to making an experimental discovery. (227)

If P were in this way problematic then Wittgenstein would deny it status as a mathematical proposition, for it then lacks an essential mark of such propositions, viz. it does not play a concept-moulding rôle.

Mathematics teaches us to operate with concepts in a new way. And hence it can be said to change the way we work with concepts.

But only a mathematical proposition that has been proved or that is assumed as a postulate does this, not a problematic proposition. (228)

The question of what Wittgenstein means by "concept" in this passage and in others where he comments on the concept-moulding function of mathematical propositions is exceedingly important - and difficult. Here I merely quote a hint as to the answer.

'Concept' is something like a picture with which one compares objects.

There is of course no sharp dividing line between language-games which work with concepts and others. What is important is that the word "concept" refers to one kind of expedient in the mechanism of language-games. (229)

The second, constructive, proof enables an intuitionist to get 'a clear view of the application' of P since it does not lack reproducibility. Whereas P seems to be the outcome of a peculiar experiment when it is 'proved' by the first proof, it can be regarded as similar to the outcome of a calculation when it is demonstrated by the second. This proof incorporates only such/

such procedures as support the ideas of correctly going through the proof and making a mistake in going through it. There is no question as to what counts as a reproduction of the proof, no room for doubt 'whether this is really the pattern of this proof', since the proof involves only procedures that are 'plain to view' in the sense that they are not, as it were, private. Agreement in the techniques to be applied in constructing the proof is possible; to master the proof is to master the techniques, and once mastered they do not become someone's techniques (like NN's deft technique with a billiard cue) any more than someone's mastery of the multiplication tables makes them his tables. In view of this possibility of agreement, P as proved by the second proof can be understood as a concept-moulding proposition - and hence as a mathematical proposition. (230)

I want to consider, in conclusion, a passage in Remarks which I find somewhat puzzling. Wittgenstein says:

One application of a mathematical proposition must always be the calculating itself. That determines the relation of the activity of calculating to the sense of mathematical propositions. (231)

In the next sentence he makes the important point that since "we judge identity and agreement by the results of our calculating" it follows that "we cannot use agreement to explain calculating." Bearing in mind that, for those who understand it, "the mathematical/

mathematical proposition determines a path... is a rule," we may perhaps reformulate these remarks in this way: Any mathematical proposition must serve as a rule, one application of which is calculating the result stated in the proposition. Such an equation as ' $146 \times 176 = 25,696$ ', to be understood as a mathematical proposition, must serve as the standard of (correctly) multiplying 146 by 176. But this is not enough, for Wittgenstein seems to cast this equation not merely in the rôle of a standard, but in the rôle of a rule, technique or procedure which is involved in multiplying 146 by 176. It is perhaps conceivable that e.g. the equation ' $2 \times 2 = 4$ ' (regarded as a rule) should be procedurally involved in multiplying 2 by 2, as well as being the standard of correctly multiplying 2 by 2. What does this mean? Simply that in calculating '2 times 2 equals...' we follow the rule implicit in ' $2 \times 2 = 4$ ' if we calculate correctly, i.e., if we get 4 as the result. In the same way there seems no reason why we should not say: in multiplying 146 by 176, the rule implicit in ' $146 \times 176 = 25,696$ ' is followed if the multiplication is correctly carried out, i.e., if 25,696 is the result. (232)

It might be suggested that to say the rule is 'followed' in such a calculation is simply to say that, as assessed by reference to the standard implicit in the equation, it is a correct calculation, and/

and that therefore equations need not be construed as procedures involved in calculations. On the other hand, Wittgenstein gives another account of calculating, wherein he asserts that 'calculating is a technique'.

The prophecy does not run, that a man will get this result when he follows this rule in making a transformation - but that he will get this result when we say that he is following the rule. What if we said that mathematical propositions were prophecies in this sense: they predict what result members of a society who have learnt this technique will get in agreement with other members of the society? ' $25 \times 25 = 625$ ' would thus mean that men, if we judge them to obey the rules of multiplication, will reach the result 625 when they multiply 25×25 . That this is a correct prediction is beyond doubt; and also that calculating is in essence founded on such predictions. That is to say, we should not call something 'calculating' if we could not make such a prophecy with certainty. This really means: calculating is a technique. And what we have said pertains to the essence of a technique. (233)

Wittgenstein's use of the word Technik is sometimes a bit puzzling: for example, he calls mathematics 'a motley of techniques of proof' and he also says "to understand a language is to be master of a technique." (234) His use of Rechnen is also puzzling: remember that 'one application of a mathematical proposition must always be the calculating itself' (das Rechnen selber), and we also read "Calculating is a phenomenon which we know from calculating. As language is a phenomenon which we know from our language." (235) At all events, I think the best way to interpret/

interpret Wittgenstein's identification of 'calculating' with 'a technique' is to say: to calculate is to engage in a practice. But that does not mean: to calculate is to follow conventions upon which people have come to agree (say, because of their usefulness). For this would be to explain calculating in terms of agreement, whereas, as Wittgenstein points out, what we call 'calculating' serves as a criterion for what we call 'agreement'. On this interpretation of the Rechnen-Technik identification, we may regard mathematical propositions as C-rules of practices: qua C-rules, however, they are not quasi-instrumentally involved in calculating as procedures and techniques (in the ordinary sense), since the latter are R-rules. Thus we say: In proceeding in accordance with (rules implicit in) mathematical propositions, we calculate; and not: In calculating, we proceed in accordance with these rules. As to the important connection between calculating qua practice and the sort of 'prophecy with certainty' mentioned in this passage, I only offer this suggestion: the very possibility of such prophecies marks the existence of practices (e.g. 'calculative practices'), and marks the concept-moulding function of their C-rules (e.g. of mathematical propositions qua C-rules of calculative practices).

[2] The relation between inference-rules - by which I mean both rules formulated in theses of the classical propositional calculus (CPC) and 'rules' implicit in determinate hypothetical sentences - and arguments or inferences is of some interest to a study of the 'application' of logic. Logical theses serve both as standards of valid (deductive) inference and as C-rules of the practice of making such inferences: in playing this dual rôle they resemble mathematical propositions which, as I suggest above, function both as standards of correct calculation and as C-rules of the practice of calculating. In other words, rules implicit in logical theses (mathematical propositions) can be regarded both as (i) applied, qua standards, to deductive inferences (calculations), and as (ii) applied, qua C-rules, in (valid) deductive inferences ((correct) calculations). As O'Connor says, "the standard of a valid argument in logic is, in an important way internal to the argument."⁽²³⁶⁾ In the following discussion I enquire into the sense in which inference-rules are 'internal' to arguments.

Ryle's contribution to the symposium 'Why Are the Calculuses of Logic and Arithmetic Applicable to Reality?' serves as starting point and guideline of this discussion. His essay is noteworthy both for a distinction among "relevant notions of application" and for/

for a clear statement of the view that inference-rules are involved in arguments and inference-performances similarly as e.g. rules of chess are involved in chess-moves. Ryle distinguishes among three basic senses of 'apply' and 'application' as relevant to the symposium-questions:

- (1) The 'specification' sense, in which "a meaty hypothetical is an application of a bony hypothetical"; e.g. 'If, given that to-day is Monday to-morrow is Tuesday, then given that to-morrow is not Tuesday to-day is not Monday' is a 'specification' of the thesis 'CCpCCNqNp'.
- (2) The 'observance' sense, in which "a person who executes an operation in accordance with a rule is said to be applying the rule"; e.g. a person who concludes that to-day is not Monday upon hearing that to-morrow is not Tuesday "is conjointly applying" both (the rule formulated in) the thesis 'CCpCCNqNp' and (the rule implicit in) the specification of this thesis: 'If, given that to-day is Monday to-morrow, &c.' (sic)
- (3) The 'fit' sense, in which a police-description of a wanted man may apply or partially apply to someone else. (Ryle distinguishes also a derivative sense, in which the description "may not be intended to apply to me".)(237)

Ryle makes this distinction because "some people have worried themselves by speculating how or why the rules of inference apply to the world", and they have worked themselves into this state of perplexity by confusing the 'observe' or 'specify' sense of 'apply' with the 'fit' sense.(238) That is, they fail to recognise that logical rules are connected with 'reality' only in so far as they are 'observed' in making and in criticising so-called 'substantial/

'substantial inferences', or in so far as they are exemplified in 'specifications'. Ryle argues that those "who construe logicians' rule-formulae as descriptions of the spine and ribs of the world" are misled by the 'fit' sense of 'apply' into "committing only a more ambitious form of the same error as that committed by those who construe these rule-formulae as premisses requiring to be intellectually acknowledged before intelligent performances can begin." (239) This error is to 'assume that a logician's rule-formula "says" something informative'. But such formulae do not, according to Ryle, 'say' something uninformative either: they are codifications of standards of deductive inference, and are hence as much 'Procrustean' performance-rules as 'the rules of the road, of chess, cricket, syllogistic reasoning and rifle-range practice'. 'Procrustean' are to be distinguished from 'canonical' performance-rules: the latter are uncodifiable and learnt by practice rather than by drill. Principles of inductive inference are comparable to canonical rules such as tactical and strategic maxims of chess. But for either,

... it is nonsense to ask how or why rules of logic apply to the world. Both the Procrustean and the canonical rules of logic are performance-rules. Only performances can be or fail to be in accordance with them. If they are applied, that is a fact about the efficiency and intelligence of theorists, not a fact about any radical docility of the world. (240)

A convenient way to elucidate the 'specification' sense of 'application' - the sense in which e.g. "If John is not unhappy then John is happy" is an 'application' of the formula 'CNNop' - will be to discuss comments on Ryle's argument by his fellow-symposiast Levy. One point on which Ryle insists, that determinate hypotheticals are not consequences of the formulae of which they are specifications, is attacked by Levy. Ryle had contended,

A logician's closed hypothetical stands to the corresponding open inference-rule not as an implication of it, but as an application of it. (241)

Before turning to Levy, however, we must remark Ryle's muddle here in confusing theses of the CPC with inference-rules formulated in them; this is not so much confusion on Ryle's part as failure to make this distinction (e.g. he later talks of 'applying the open hypothetical' when he surely means to say 'applying the open inference-rule'). In itself this is a trivial point for, as Popper points out in his contribution to the symposium, every thesis of the CPC can be interpreted as an inference-rule. (242) But in view of Ryle's distinction between the 'specification' and 'observance' senses of 'application', it would seem to be important for him to distinguish carefully between formulae and rules. The latter are 'observed' in inference-performances, whereas determinate hypotheticals stand to the former as 'specifications', and not as 'deductions'.

Lewy, on the other hand, holds that "by explaining this use of 'application' we have also explained one use of 'consequence': for in one of its uses, the expression 'P is a consequence of Q' means, I think, precisely that P is an application of Q, in this sense of application,"⁽²⁴³⁾ His general formulation of necessary and sufficient conditions for P to be an application ('specification') of principle Q is:

there is a propositional schema F, such that (1) P is a value of F, and (2) Q asserts with regard to any proposition R that if R is a value of F then R is true.

Notice that Lewy speaks of Q both as a principle and as an assertion. He holds that theses of the CPC are to be understood as universally quantified propositions which assert the (logical) truth of all values of some propositional schema. For example, the determinate hypothetical "If John is not unhappy then John is happy" would be an application ('specification') of the thesis ' $\Pi p \text{CNNpp}$ ' since ' CNNqq ' is a propositional schema of which "If John is not unhappy then John is happy" is a value, and ' $\Pi p \text{CNNpp}$ ' asserts, according to Lewy, that any value of ' CNNqq ' is (logically) true. Hence, Lewy argues, we can say that "If John is not unhappy then John is happy" follows from ' $\Pi p \text{CNNpp}$ '.

Against Lewy's claim I offer the following objections. (i)

It/

It is not clear to me that theses of the CPC 'assert' what Levy says they do. Doesn't ' $\neg p \supset \neg \neg p$ ', for example, 'assert' simply that, for any p , not-not- p implies p ? (ii) Even if thesis Q were to 'assert' that all values of propositional schema F are (logically) true, it is false to say that P follows from Q , for P follows from Q and a statement that P is a value of F . (iii) It might be urged that theses of the CPC do not 'assert' that all values of a propositional schema are (logically) true, since this is something that the thesis 'shows' but does not 'say'.

(iv) What I think Levy has in mind can be formulated in terms of the notion of instantiation. To say that "If John is not unhappy then John is happy" is an application ('specification') of ' $\neg p \supset \neg \neg p$ ', and hence a consequence of this thesis, is just to apply a rule of universal instantiation to the universally quantified thesis, substituting "John is happy" for ' p '. The question arises, however, whether the rule of substitution for the CPC warrants such an inference. Some logicians have thought that it does. Tarski, for example, formulates the rule of substitution so as to allow replacement of sentential variables by "other sentential variables or by sentential functions or by sentences." (244) And Prior writes that the sentence "It is either true/

true or false that Geronimus is dead" instantiates the thesis 'For any p , it is either true or false that p '. (245) We ought to distinguish, however, between, on the one hand, replacing propositional variables in theses of the CPC by well-formed formulae (wff) and, on the other hand, substituting sentences of some natural language for these variables. Assuming for the moment that theses of the CPC are universally quantified propositional schemata, then the distinction I have in mind can be illustrated by comparing universal instantiation in functional or class logic with substitution in the CPC. Following J.L. Mackie, I distinguish between 'U.I. (arb)':

From $\vdash (\underline{x}) \underline{F}\underline{x}$ can be inferred $\vdash \underline{F}\underline{y}$, for \underline{y} an arbitrarily selected individual;

and what I call simply 'U.I.':

From $\vdash (\underline{x}) \underline{F}\underline{x}$ can be inferred $\vdash \underline{F}\underline{a}$, for \underline{a} a 'real' individual. (246)

That is, U.I. (arb) warrants deduction of ' $\underline{F}\underline{y}$ ' from ' $(\underline{x}) \underline{F}\underline{x}$ ', for ' \underline{y} ' an individual variable; whereas U.I. warrants deduction of ' $\underline{F}\underline{a}$ ' from ' $(\underline{x}) \underline{F}\underline{x}$ ', for ' \underline{a} ' an individual constant or definite description. This distinction is built into most formulations of Universal Instantiation. Church, for example, who employs the notion of substitution, gives U.I. the status of an axiom scheme.

*306. $(\underline{a}) \underline{A} \supset S_{\underline{b}}^{\underline{a}} \underline{A}$, where \underline{a} is an individual variable, \underline{b} is an individual variable or an individual constant, and no free occurrence of \underline{a} in \underline{A} is in a wf part of \underline{A} of the form $(\underline{b}) \underline{C}$. ($S_{\underline{b}}^{\underline{a}}$ stands for the result of substituting \underline{b} for all free occurrences of \underline{a} in \underline{A} .) (247)

Quine formulates V.I. in a metatheorem which he says "embodies the principle of application or specification - the principle that leads from a general law, a universal quantification $(\alpha)\phi$, to each special case χ falling under the general law." (249)

*231. If χ is like ϕ except for containing free occurrences of ζ wherever ϕ contains free occurrences of α , then $\vdash (\alpha)\phi \supset \chi$.

In proving this Quine establishes two cases, for ζ a variable and for ζ an 'abstract' (e.g. a definite description). Now we might make a similar distinction within the rule of substitution for the CFC, i.e. between replacing propositional variables by wff and replacing them by verbal sentences. For example, we could introduce the words "[any wff] or proposition-expressing sentence of some natural language" into Church's statement of the rule of substitution:

*201. From A , if b is a variable, to infer A/b (1) may be any wff, and A/b results from substitution of b for each occurrence of b throughout A . (249)

In this way we would make *201. analogous to *306., and bring Church's version of the rule of substitution into line with the thinking of Tarski and Prior.

In so far as Levy's claim - that determinate hypotheticals which are 'specifications' of logical theses can be said to follow from theses/

theses - rests on this analogy between substitution in the CPC and instantiation in class and functional logic, then by attacking the analogy one may argue against the claim. I do not think that theses of the CPC are usefully to be regarded as schemata universally quantified over the range of propositions, and hence it seems to me that substitution of assertible sentences for propositional variables is not logically warranted analogously as substitution of individual constants for individual variables; in other words, whereas both 'Fa' and 'Fy' may be said to be deducible from '(x)Fx', only e.g. 'CNNKpaKpa', and not "If John is not unhappy then John is happy", may be said to be deducible from the thesis 'CNNpp'. It follows that Levy's claim, if based on this analogy, does not hold. (a) If theses are to be viewed as universally quantified propositional schemata, then surely their range of quantification consists of truth-values, and not of propositions - neither wffs nor assertible sentences of a natural language. Thus, for example, 'IpCNNpp' should be construed as: 'CNN11 = 1 & CNNoo = 1'; and 'IpIpqCpCqp' can be interpreted: 'C1C11 = 1 & C1C01 = 1 & C0C10 = 1 & C0C00 = 1'. In this way, quantification in the CPC would resemble quantification over a specified range of individuals in functional and class logic. (b) But why should quantification be introduced into the CPC at all? In/

In some versions of the CPC, e.g. Russell's Theory of Implication, and in some non-classical propositional calculi, e.g. Lukasiewicz-Tarski's 'extended sentential calculus', quantification is legitimately introduced for definitional purposes: (250) but if, as in other versions of the CPC, these purposes are served without need of quantification, then why should theses of these calculi be construed as universally quantified? Levy would argue that propositional schemata are not, unless quantified, propositions: but surely the way to indicate that a schema has thesis-status - where the schema is developed within a quantificationless CPC - is to prefix the thesis-sign '⊢' to it. (251)

Turning now to the second sense of 'apply' distinguished by Ryle - the sense in which inference-rules are said to be 'applied' ('observed') in inference-performances - it is again convenient to begin by considering Levy's comments on Ryle's argument. Levy points out that Ryle goes wrong in saying that someone who argues: 'Tomorrow is not Tuesday, so today is not Monday', is "conjointly applying" (the rules formulated in) both 'CCpCNCqNp' and 'If, given that today is Monday tomorrow is Tuesday, then given that tomorrow is not Tuesday today is not Monday'; as Levy says, someone "conjointly applying" these rules would argue: 'Tomorrow is/

is Tuesday on condition that today is Monday, so today is not Monday on condition that tomorrow is not Tuesday'; whereas, in arguing: 'Tomorrow is not Tuesday, so today is not Monday', one 'applies' only the 'rule' implicit in 'If tomorrow is not Tuesday then today is not Monday'.⁽²⁵²⁾ Lewy then combines the 'specification' and 'observance' senses of 'application' into a principle which I paraphrase:

- (I) An inference 'A, so B' is an observance of the rule formulated in thesis ' $C \propto \beta$ ' of the GPC (α and β are wff) if, and only if, the determinate hypothetical 'If A then B' is a specification of ' $C \propto \beta$ '.⁽²⁵³⁾

It will be observed that (I) accounts ^{only} for 'observance' of rules formulated in logical theses. I want now to develop an analogous principle for 'rules' implicit in determinate hypotheticals which are not (necessarily) 'specifications' of some logical thesis. To implement my discussion I consider Ryle's analysis, in "If", "So", and "Because", of determinate hypotheticals.

Ryle claims, at one point in this essay, to "harden the edges of the notion of application."⁽²⁵⁴⁾ He means 'application' in the 'observance' sense; though, as we shall see, he uses 'apply' intransitively (i.e. in the 'fit' sense) in one passage. Arguing that both "the officially recognised Rules of Inference" as well as "the most 'meaty' and determinate hypothetical statements" are involved as principles rather than as premises in/

in inferences, Ryle continues:

The argument "Today is Monday so tomorrow is Tuesday" is an application of "if today is Monday, tomorrow is Tuesday"; and it is in this notion of application that lies the answer to our question "How does a valid argument require the truth of the corresponding hypothetical statement?" (255)

A few pages later we find him using 'apply' in a somewhat peculiar way:

... just as the inference "p, so q" does not embody "if p, then q" as a component of its premiss, but rather applies it in being an operation with "p" and "q" executed in conformity with it, so "q, because p" does not embody "if p, then q" as part of its "because" clause but applies it in another way. (256)

When Ryle says that an inference 'applies' an hypothetical statement, he must surely mean that someone making the inference 'observes' a rule formulated in the hypothetical. Note that he calls inference 'an operation': Ryle holds that 'theorising conduct' is like 'practical conduct' in that it involves rules, techniques, skills, &c. He writes, for example: "in arguing (and following arguments) a person is operating with a technique or method, i.e. he is exercising a skill; but in making or considering hypothetical statements he is, for example, giving or taking instruction in that technique or operation." (257) (This should be compared with the distinction between Procrustean and canonical rules: "methods, techniques, crafts, skills, etc.," being uncodifiable/

uncodifiable and learnt by practice, "are subject to canonical rules"; but since inference-rules are codifiable into theses or hypotheticals, and therefore can be conned by drill rather than by practice, it follows that they are Procrustean rules and not - as Ryle implies in this passage - canonical rules.)

The notion of 'statement-specification' must next be introduced. Ryle writes:

It is because hypothetical statements embody statement specifications, that an inference from one statement to another can be described as being "in accordance with" or being "an application of" the hypothetical. (258)

In explaining how determinate hypotheticals are related to statements which satisfy or fulfil their statement-specifications, Ryle uses 'apply' in its intransitive, or 'fit' sense:

That something is an eligible filling for (or satisfies) an open specification is part of what is meant by saying that the statement, rule, or warrant (etc.) incorporating the specification "applies" to that something. (259)

Consider, for example, the hypothetical sentence "If A then B": Ryle claims that in stating 'if A then B' (i.e. in asserting B on condition that A), one states neither that A nor that B; whereas in arguing or inferring in accordance with the 'rule' implicit in 'If A then B', one asserts B on assertible (if not asserted) grounds that A. Presumably, Ryle would say that this 'rule' "applies" to these assertions, since the hypothetical 'If A then B' is satisfied by/

by statements that A and that B. He compares determinate hypotheticals with such 'variable hypotheticals' as 'For all x , if x is a man, x is mortal', claiming that "some kind of openness, variableness, or satisfiability characterises all hypothetical statements alike." (260)

To emphasise the 'openness' of determinate hypotheticals Ryle points out that we can reword e.g. 'If today is Monday then tomorrow is Tuesday' either modally ('It cannot be Monday today and not be Tuesday tomorrow') or subjunctively ('If today be Monday then tomorrow be Tuesday'): because such reformulations do not, unlike the hypothetical 'If today is Monday, &c.', embody assertible sentences, Ryle suggests that their possibility supports his analysis of determinate hypothetical statements. His critics, on the other hand, argue that neither comparison with quantified hypotheticals (which are not, in any case, 'open'), nor the possibility of modal and subjunctive reformulations of hypothetical sentences, serves to demonstrate that hypothetical statements (i.e. asserting B on condition that A) consist, not of (categorical) statements, but of (utterance of) statement-specifications. (261)

(It will be evident, from my terminology here, that the sentence/statement distinction is crucial in this discussion. Ryle does not make this clear, and what he calls 'hypothetical statements' I have taken to be 'hypothetical sentences', except when I speak of 'asserting... on condition that...'. This locution is borrowed from von Wright.) (262)

Assuming/

Assuming, for the moment, that determinate hypotheticals contain statement-specifications, let us connect the notion of 'statement-specification' with the view that 'rules' implicit in determinate hypotheticals are observed in inferences. First we introduce the idea of 'inference-specification' ('I-specification'). A determinate hypothetical, H , is associated with statement-pairs, (p_k, q_k) , where p_k satisfies H 's protasis statement-specification and q_k satisfies its apodosis specification, such that q_k is asserted on grounds which, if asserted, would be asserted in p_k : any such (p_k, q_k) is said to be an I-specification of H . Thus, for example, a statement-pair made up of statements that today is Monday and that tomorrow is Tuesday is an I-specification of the hypothetical 'If today is Monday then tomorrow is Tuesday', in so far as the statements are termini of an inference made in accordance with the 'rule' implicit in this hypothetical. I-specifications need not consist of sentential utterances. A statement to the effect that tomorrow is Tuesday might be e.g. a nod in response to the question 'Is tomorrow Tuesday?'; if the head-nodder replies to a further question 'Do you "say" tomorrow is Tuesday because you believe today to be Monday?' with another affirmative nod, then the two nods may be said to constitute an I-specification of 'If today is Monday then tomorrow is Tuesday'. In the light of this analysis we see the/

the point of Ryle's remark that an inference is describable as being 'in accordance with' (a 'rule' implicit in) an hypothetical because determinate hypotheticals embody statement-specifications. We also see that determinate hypotheticals can be said to embody statement-specifications because inferences are 'in accordance with' ('rules' implicit in) hypotheticals. This equivalence can be expressed in an analogue of principle (I):

- (II) An inference 'A, so B' is an observance of the 'rule' implicit in determinate hypothetical H if, and only if, (A,B) is an I-specification of H.

Finally, we state (I) in terms of (II):

- (III) An inference 'A, so B' is an observance of the rule formulated in thesis ' $C \propto \beta$ ' of the CPC if, and only if, (A,B) is an I-specification of some determinate hypothetical which is a specification of ' $C \propto \beta$ '.

NOTES.

- (1) 'To put into practice or operation, to carry into action or effect'.
- (2) 'On Using Language', The Philosophical Quarterly (1956), pp. 334-5.
- (3) Thinking and Meaning (London: H.K. Lewis, 1947), p.5.
Cf. also pp. 6-7.
- (4) Ibid., p.5.
- (5) Cf. e.g. Theaetetus, 184c-186.
- (6) De Anima, III, 8.
- (7) Knowledge and Perception (Oxford, 1950), p. 106.
- (8) 'Knowing How and Knowing That', Proc. Aristotelian Soc. (1946), p. 16.
- (9) (a) Prior, Formal Logic (Oxford, 1953), p. 106.
(b) Urmson, 'On Grading', reprinted in Logic and Language, 2d ser., ed. by Flew (Oxford: Blackwell, 1953), p. 185.
(c) Rhine, 'A Concept-Formation Approach to Attitude Acquisition', Psych. Rev. (1958), p. 362.
(d) De Morgan, Formal Logic (London: Taylor and Walton, 1847), p. 280.
(e) Price, Thinking and Experience (London: Hutchinson, 1953), p. 288.
(f) Russell, The Problems of Philosophy (Oxford, 1912), p. 70
- (10) Urmson, op. cit., p. 162.
- (11) Ibid., p. 173.
- (12) Ibid., pp. 175, 177, 179, 183, 185.
- (12a) Cf. Black, 'Notes on the Meaning of "Rule"', Theoria (1958) pp. 107-126.
- (13)/

- (13) Cf. Kritik der reinen Vernunft, A179/B222, A509/B537, &c.
- (14) The Moral Point of View (Cornell, 1958), p. 126. Cf. also O'Connor, 'Validity and Standards', Proc. Aristotelian Soc. (1957), p. 226.
- (15) For a discussion of this point, cf. Hanson, Patterns of Discovery (Cambridge, 1958), ch. I; and Urmson, 'Recognition', Proc. Aristotelian Soc. (1956), pp. 271-2.
- (16) 'Two Concepts of Rules', Philosophical Review (1955), p. 3. Cf. also pp. 24-29.
- (17) Cf. Opusc. I, de Operationibus occultis Naturae ad quendam militem ultzramontanum, in Philosophical Texts, ed. and transl. by Gilby (Oxford, 1956), par. 43. Cf. also par. 124.
- (18) A Behavior System (Yale, 1952), p. 327.
- (19) Intention (Oxford: Blackwell, 1957), par. 32. Cf. also par. 8.
- (20) Ibid., p. 55.
- (21) Ibid., p. 67.
- (22) Ibid., p. 14.
- (23) 'Why Are the Calculuses of Logic and Arithmetic Applicable to Reality?', Proc. Aristotelian Soc. Suppl. (1946), p. 28. Cf. also The Concept of Mind (London: Hutchinson, 1949), ch. II.
- (24) Philosophical Investigations, transl. by Anscombe (Oxford: Blackwell, 1953), par. 564.
- (25) The Concept of Mind, op. cit., p. 48. It is likely that Ryle is using "efficiently" in a secondary sense here, i.e. in a loose strict sense than that of my usage.
- (26) H.M. Adams uses 'double barrelled' in his note 'Mr. Hare on the Role of Principles in Deciding' (Mind, 1956); what he calls "double barrelled decisions" lead to infinite regress, but he does not distinguish between types of regress.
- (27) "Über den Gemeinspruch: Das mag in der Theorie richtig sein, taugt aber nicht für die Praxis", Kants Werke VIII (Akademieausgabe), p. 275.

- (28) Ibid., p. 275 "... zu dem Verstandesbegriffe, welcher die Regel enthält, muss ein Aktus der Urteilkraft hinzukommen, ^{wedurch} ~~Praktik~~ unterscheidet, ob etwas der Fall der Regel sei oder nicht; und da für die Urteilkraft nicht immer wiederum Regeln gegeben werden können, wonach sie sich in der Subsumtion zu richten habe (weil das ins Unendliche gehen würde), so kann es Theoretiker geben, die in ihrem Leben nie praktisch werden können, weil es ihnen an Urteilkraft fehlt."
- (29) Kritik..., op. cit., A133/B172.
- (30) Kants Werke V, p. 169. "... zu einer objektiven [Regel], der sie [sc. die Urteilkraft] ihr Urteil anpassen kann, weil dazu wiederum eine andere Urteilkraft erforderlich sein würde, um unterscheiden zu können, ob es der Fall der Regel sei oder nicht."
- (31) Ibid., p. 179.
- (32) Kritik..., op. cit., A133/B172.
- (33) Analytica Posteriora, I, 34, transl. by Mure (Oxford, 1928).
- (34) 'Knowing How...', op. cit., p. 2.
- (35) The Concept..., op. cit., p. 30. Cf. also 'Why Are the Calculuses...', op. cit., p. 27.
- (36) The Concept..., p. 31.
- (37) 'Knowing How...', pp. 2-3.
- (38) Cf. Kritik..., op. cit., A138/B177.
- (39) 'Intelligence and Intelligent Conduct', Proc. Aristotelian Soc. (1948), pp. 187-204.
- (40) Ibid., p. 190.
- (41) Ibid., p. 191.
- (42) Ibid., p. 192.
- (43) Thinking and Experience, op. cit., p. 59.
- (44) Ibid., p. 78. Cf. also pp. 52-74 for Price's discussion of 'primary recognition' as "an innate or unlearned capacity."
- (45)/

- (45) Cf. Ryle, 'Sensation', in Contemporary British Philosophy, 3d ser., ed. by H.D. Lewis (London: Allen and Unwin, 1956), pp 427-443. "Perceptual recognition [involves] the possession and exploitation of knowledge previously acquired." (p. 438, my italics); "Perceiving... is exercising an acquired skill." (p. 440).
- (46) The Blue and Brown Books (Oxford: Blackwell, 1958), p. 143.
- (47) Ibid., p. 12.
- (48) Ibid., p. 14.
- (49) Ibid., p. 14.
- (50) Ibid., p. 15.
- (51) Ibid., pp. 12-13.
- (52) Ibid., p. 13.
- (53) Ibid., p. 13.
- (54) Cf. ibid., p. 42, where the 'rule of thumb' is stated: "If you are puzzled about the nature of thought, belief, knowledge, and the like, substitute for the thought the expression of thought, etc."
- (55) Cf. Grundlegung zur Metaphysik der Sitten, in Kants Werke IV, pp. 397-398.
- (56) Philosophical Investigations, op. cit., par. 198.
- (57) Ibid., par. 201.
- (58) Ibid., par. 201. "... es eine Auffassung einer Regel gibt, die nicht eine Deutung ist; sondern sich, von Fall zu Fall der Anwendung, in dem aussert, was wir 'der Regel folgen', und was wir 'ihr entgegenhandeln' nennen."
- (59) Ibid., par. 198.
- (60) (a) Ibid., par. 84.
- (b) Ibid., par. 85.
- (c) Ibid., par. 87.
- (61)/

- (61) Ibid., par. 86.
- (62) The Blue and Brown Books, op. cit., pp. 14-15.
- (63) Ibid., p. 14.
- (64) Ibid., p. 15.
- (65) Ibid., p. 14.
- (66) Ibid., p. 15.
- (67) Ibid., p. 143. Cf. also Investigations, par. 146.
- (68) Ibid., p. 141. Cf. also Investigations, pars. 185-189.
- (69) Ibid., p. 142. Cf. Investigations, par. 188.
- (70) Ibid., p. 142.
- (71) Ibid., pp. 142-143.
- (72) Ibid., p. 143.
- (73) Ibid., p. 143. Cf. Investigations, par. 186.
- (74) Ibid., p. 143.
- (74a) Philosophical Investigations, op. cit., par. 238. "Damit es mir erscheinen kann, als hatte die Regel alle ihre Folgesätze zum Voraus erzeugt, müssen sie mir selbstverständlich sein."
- (75) The Blue and Brown Books, op. cit., p. 143.
- (76) Philosophical Investigations, op. cit., par. 241.
- (77) Ibid., par. 223.
- (78) Ibid., par. 219.
- (79) Ibid., par. 217.
- (80) "'Good At'", Proc. Aristotelian Soc. Suppl. (1958), p. 193.
- (81) 'Contingence in the Philosophy of Leibniz', Philosophical Review (1952), pp. 26-39.
- (82)/

(82) Ibid., p. 36.

(83) Ibid., pp. 37-38.

(84) Ibid., p. 38.

(84a) Another point in Leibniz bears on this discussion of regress, in particular on my procedure- and justification-regresses. Leibniz writes, in his fifth letter to Clarke, "For a man never has a sufficient reason to act, when he has not also a sufficient reason to act in a certain particular manner... Wherefore, when there is a sufficient reason to do any particular thing, there is also a sufficient reason to do it in a particular manner..." (par. 17; but cf. par. 66, where this is said to be true only of God's acts). Both the procedure- and the justification-regresses are easily derived from Leibniz's dictum. If one has a sufficient reason for doing x in manner m, then he must have a sufficient reason for doing x in manner m in manner n, and so on.

(85) (I) (a) Goodman, Fact, Fiction and Forecast (London: Athlone, 1954), p. 45.

(b) De Morgan, 'On the Syllogism, No. V', Cambridge Philosophical Transactions (Vol. X, 1864), p. 454.

(c) Mill, A System of Logic (London, 1875), Bk. I, ch. ii, 84.

(d) Morris, 'Foundations of the Theory of Signs', International Encyclopedia of Unified Science I, No. 2 (1938), p. 7.

(II) (a) Kant, Kritik..., op. cit., A138/B177. "... wie reine Verstandesbegriffe auf Erscheinungen überhaupt angewandt werden können." (Kemp Smith (mis)translates "... can be applicable...")

(b) Grant, '"Good AT"', op. cit., p. 188.

(c) Mayo, Ethics and the Moral Life (London: Macmillan, 1958) p. 26.

(d) Baier, op. cit., p. 192.

(e) Kelsen, General Theory of Law and State, transl. by Wedberg (Harvard, 1946), p. 128.

(III)/

- (III) (a) Braithwaite, Scientific Explanation (Cambridge, 1953), p. 13.
- (b) Feyerabend, 'Complementarity', Proc. Aristotelian Soc. Suppl. (1958), p. 87.
- (c) Williams, The Ground of Induction (Harvard, 1947), p. 28.
- (d) Paul, 'G. E. Moore', in The Revolution in Philosophy (London: Macmillan, 1956), p. 66.
- (e) Warnock, Berkeley (London: Penguin, 1955), p. 220.
- (f) Mayo, op. cit., p. 10.
- (86) Individuals (London: Methuen, 1959), p. 139.
- (87) (a) Chisholm, 'Sentences About Believing', Proc. Aristotelian Soc. (1956), p. 128.
- (b) O'Connor, 'Names and Universals', Proc. Aristotelian Soc. (1953), p. 137.
- (c) Ayer, 'Individuals', reprinted in Philosophical Essays (London: Macmillan, 1954), p. 9.
- (88) Essay Concerning Human Understanding, Bk. III, ch. iii, sec. 3.
- (89) Ibid., Bk. III, ch. i, sec. 6.
- (90) Ibid., Bk. III, ch. ii, sec. 2.
- (91) Ibid., Bk. III, ch. ii, sec. 6.
- (92) Mill, loc. cit.
- (93) Ibid., Bk. I, ch. ii, 5.
- (94) De Morgan, 'On the Syllogism, No. V', op. cit., pp. 454-455.
- (95) Peirce, Collected Papers II (Harvard, 1932), pars. 431-434.
- (96)/

- (96) 'Über Sinn und Bedeutung', Zeitschrift für Philosophie (1892), p. 31. Transl. and reprinted in Philosophical Writings of Gottlob Frege, ed. by Geach and Black (Oxford: Blackwell, 1952), p. 61. "Ein Eigennamen (Wort, Zeichen, Zeichenverbindung, Ausdruck) drückt aus seinen Sinn, bedeutet oder bezeichnet seine Bedeutung. Wir drücken mit einem Zeichen dessen Sinn aus und bezeichnen mit ihm dessen Bedeutung." Note that Frege speaks of our expressing its (i.e. the sign's) sense, and of our designating its reference.
- (97) One logician who mentions this distinction, though not precisely in these terms, is W.E. Johnson. He writes: "There is some danger of confusing two different uses of the verb 'to characterise', which may be partly responsible for the historical dispute concerning the relation of particular to universal. ... 'such and such a quality of adjective characterises such or such an object or substantive' [and] 'the thinker characterises such or such an object', [in which] 'characterises' means 'cognitively determines the character of.'" (Logic I, pp. 11-12)
- (98) An Analysis of Knowledge and Valuation (LaSalle: Open Court, 1946), p. 39 (my italics).
- (99) Ibid., p. 39 (my italics).
- (100) "Now the temptation to say that he [sc. someone who "asks what the meaning of a word is"] is asking what the word refers to arises from the fact that the job of a great many words is that of referring to something, so that to ask how the word is used and to ask what it refers to come to the same thing." (Nowell-Smith, Ethics (London: Penguin, 1954), p. 66).
- (100a) Cf. Ayer, 'Individuals', op. cit., p. 10; "To determine which are the occasions to which they can apply..."; Ryle, 'Meaning and Necessity', Philosophy (1949), p. 72; "... a senseless designator cannot... apply to anything."
- (101) 'Properties and Classes', Proc. Aristotelian Soc. (1958), p. 33. (my italic)
- (102) Ibid., p. 36.
- (103) Ibid., p. 38.
- (104) Cf. ibid., p. 46. "So the use of classificatory words is not just the same thing as the existence of natural classes, it does really presuppose it."
- (105)/

- (105) 'On Referring', reprinted in Essays in Conceptual Analysis, ed. by Flew (London: Macmillan, 1956), p. 40.
- (106) Ibid., pp. 41-42 (my italics). Cf. also Introduction to Logical Theory (London: Methuen, 1952), pp. 145-146.
- (107) The Blue and Brown Books, op. cit., pp. 56-57 (my italics).
- (108) Ibid., p. 58.
- (109) 'The Use of Sentences', Analysis (1956), p. 3.
- (110) '"What-for" Questions and the Use of Sentences', Analysis (1957), p. 87.
- (111) Ibid., p. 91.
- (112) Ibid., p. 89.
- (113) 'On Referring', op. cit., pp. 30, 31.
- (114) 'On Meaning and Verification', Mind (1953), p. 8.
- (115) L.J. Cohen, 'On the Use of "the Use of"', Philosophy (1955), pp. 7-14; Grant, 'On Using Language', op. cit.
- (116) Verbal Behavior (New York: Appleton-Century-Crofts, 1957), p. 7.
- (117) L.J. Cohen, op. cit., pp. 8-9.
- (118) Mental Acts (London: Routledge & Kegan Paul, 1957), p. 13.
- (119) Ibid., p. 14.
- (120) Ibid., p. 36. "A part of a sighted man's colour concepts is his ability to apply them to visual experiences."
- (121) Ibid., p. 41.
- (122) Ibid., p. 40.
- (123) Ibid., p. 77.
- (124) Ibid., p. 64.
- (125)/

- (125) Ibid., p. 65.
- (126) Ibid., pp. 122-123.
- (127) Ibid., p. 124.
- (128) Ibid., p. 125.
- (129) Kritik..., op. cit., A68/B93.
- (130) (a) Ibid., A68/B93.
 (b) Ibid., A68-69/B93.
- (131) Ibid., B146. "Die Kategorie hat keinen andern Gebrauch zum Erkenntniss der Dinge, als ihre Anwendung auf Gegenstände der Erfahrung".
- (132) Ibid., B150. (transl. by Kemp Smith; unless otherwise noted, subsequent quotations are from the Kemp Smith translation.)
- (133) Ibid., B147.
- (134) Ibid., A69/B94.
- (135) Ibid., A85/B117.
- (136) Ibid., A88/B120.
- (137) Ibid., A135/B174-175.
- (138) Ibid., A136/B175.
- (139) Ibid., A138/B177. (My translations Kemp Smith does not translate "mithin" ("thereby"), and mistranslates "angewandt werden können" (cf. Note 85).)
- (140) (a) Ibid., A146/B185.
 (b) Ibid., A146/B186. Cf. also A138/B177, where Kant refers to "das transzendente Schema" as "diese vermittelnde Vorstellung".
- (141) Cf. Berkeley, Alciphron (1st and 2nd editions) Dial. VII, sect. 7., in Works, ed. by Jessop (London: Nelson, 1950), p. 334; Hume, A Treatise of Human Nature, Bk. I, pt. i, sect. 7. (Selby-Bigge edn., pp. 20, 21.); Reid, Essays on the Intellectual Powers of Man, Essay VI, ch.1. (Hamilton edn., pp. 417-418.)

- (142) Conceptual Thinking (Cambridge, 1955), p. 14. Cf. also pp. 23, 217ff.
- (143) Urmson, op. cit., p. 171.
- (144) Baier, op. cit., p. 73.
- (145) Cf. ibid., pp. 72-77. Cf. also Matthews, "Evaluative and Descriptive", Mind (1958), p. 341, where the question of "an infinite regress of criteria for criteria" is mentioned.
- (146) Grant, "Good At", op. cit., p. 188.
- (147) O'Connor, 'Validity and Standards', op. cit., p. 224.
- (148) 'Theory and Definition in Jurisprudence', Proc. Aristotelian Soc. Suppl. (1955), p. 258.
- (149) Kelsen, op. cit., pp. 60-61.
- (150) Ibid., pp. 61-62.
- (151) 'Theory and Definition in Jurisprudence', op. cit., p. 234.
- (151a) "What is it to apply a rule? At a time when philosophers are inclined to find in 'rules' and their 'application' the solution to many puzzles it is salutary to consider the difficulties of answering this question even in the legal sphere where these expressions are used in their original and least metaphorical sense" (Hart, in Mind (1951), p. 270, *my italics*). But 'following a rule' and 'applying a law' are by no means the same thing.
- (152) Baier, op. cit., p. 132. Cf. also p. 126.
- (153) 'Theory and Definition in Jurisprudence', op. cit., p. 252.
- (154) The Poverty of Historicism (London: Routledge & Kegan Paul, 1958) p. 124.
- (155) The Nature of Historical Explanation (Oxford, 1952), p. 2. Cf. also Hempel, 'The Function of General Laws in History', reprinted in Readings in Philosophical Analysis, ed. by Feigl and Sellars (New York: Appleton-Century-Crofts, 1949), p. 462.
- (156)/

- (156) 'Three Views Concerning Human Knowledge', in Contemporary British Philosophy, 3d ser., op. cit., p. 378.
- (157) Cf. Braithwaite, op. cit., pp. 13-14.
- (158) Quoted from Hanson, op. cit., p. 209. Though these are not, so far as I can ascertain, Toulmin's words - Hanson cites The Philosophy of Science (London: Hutchinson, 1953) without giving a page reference - yet Toulmin comes close to saying this on pp. 42, 84-85, 157. But cf. also p. 159 where, as Nagel points out in his review of Toulmin's book (in Mind (1954), p. 406), something is said to be inferred from atomic theory.
- (159) Laws and Explanation in History (Oxford, 1957), p. 80. Cf. also Toulmin, op. cit., pp. 34-35.
- (160) Hanson, op. cit., p. 121. Cf. also p. 90.
- (161) Ibid., p. 125.
- (162) Popper, 'Three Views Concerning Human Knowledge', op. cit. p. 361. Cf. Toulmin, op. cit., p. 108. "The problem of applying the theoretical calculus remains in physics the central problem, for a science is nothing if its laws are never used to explain or predict anything."
- (163) Ibid., p. 362.
- (164) Ibid., p. 377.
- (165) Ibid., p. 366.
- (166) Toulmin, op. cit., pp. 88-89. But cf. p. 55, where Toulmin remarks "how different are scientific explanations of the physical type from anything we could ordinarily speak of as descriptions."
- (167) Cf. Hanson, op. cit. Ch. V, for an effective critique of 'single-valued' answers to the question 'What is the logical status of the laws of classical particle physics?'.
- (168) Toulmin, op. cit., p. 78.
- (169) Ibid., p. 86.
- (170)/

- (170) Ibid., p. 89.
- (171) Ibid., p. 31.
- (172) Ibid., p. 92.
- (173) Ibid., p. 94.
- (174) Ibid., p. 112.
- (175) Ibid., p. 148.
- (176) Popper, 'Three Views Concerning Human Knowledge', op. cit., pp. 379-380.
- (176a) Ibid., p. 381.
- (177) Ibid., p. 380.
- (178) 'Der Begriff "Abgeschlossene Theorie"', Dialectica (vol. 2), pp. 333-334. "Die klassische Mechanik... ist überall eine streng "richtige" Beschreibung der Natur, wo ihre Begriffe angewandt werden können." ...nur deuten wir durch den Zusatz 'wo ihre Begriffe angewandt werden können' an, dass wir den Anwendungsbereich der Newton'schen Theorie für beschränkt halten."
- (179) (a) Kritik..., op. cit., A165/B206
- (b) Grundlagen der Arithmetik, transl. by Austin (Oxford: Blackwell, 1953), p. 99^e.
- (c) Language, Truth and Logic, 2d edn. (London: Gollancz, 1946), p. 82.
- (d) 'The Paragon of Knowledge', Philosophy (1954), p. 227.
(A good example of confusion due to the transitive/intransitive ambiguity of 'apply'.)
- (e) 'Mathematical Formalisms and their Realizations', Philosophy (1952), p. 142.
- (180) The Logic of Modern Physics (New York: Macmillan, 1927), p. 64.
- (181) 'Why Are the Calculuses of Logic and Arithmetic Applicable to Reality?', op. cit., p. 54.
- (182)/

- (182) Language, Truth and Logic, loc. cit.
- (183) 'Foundations of Logic and Mathematics', International Encyclopedia of Unified Science I, No. 3 (1939), p. 47.
- (184) An Essay on Metaphysics (Oxford, 1940), p. 254.
- (185) Grundlagen..., op. cit., p. 13^e.
- (186) 'Why Are the Calculuses...?', op. cit., p. 55.
- (187) 'Mathematical Statements - A Reply to Mr. Hirst', Philosophical Quarterly (1955), p. 153.
- (188) 'Analytic Truths', Mind (1956), p. 363.
- (189) Ibid., pp. 363-364.
- (189a) Grundlagen..., op. cit., p. 99^e.
- (190) Grundgesetze der Arithmetik II, par. 91, transl. and reprinted in Philosophical Writings of Gottlob Frege, op. cit., p. 187.
- (191) Ibid., par. 89, p. 185.
- (191a) Ibid., par. 92, p. 187. Subsequent quotations in this paragraph are also from par. 92, pp. 187-188.
- (192) Ibid., par. 88, p. 184.
- (193) Ibid., par. 105, p. 200.
- (194) Remarks on the Foundations of Mathematics, transl. by Anscombe (Oxford: Blackwell, 1956), V-13 (i.e. Part V, par. 13).
- (195) Ibid., II-4.
- (196) Grundlagen..., op. cit., p. 99^e. Another example: "The applicability of mathematical truths to the world we know in experience depends on the fact that both in mathematics and in the world of fact logical laws have absolute validity." (Walsh, Reason and Experience (Oxford, 1947), p. 93)
- (197) Einführung in das Mathematische Denken (Wien: Gerold, 1936), p. 184 (my italics), "... löst man sie ganz von der Anwendung los, trennt man all die Fäden durch, welche sie mit unserer Wortsprache verbinden, dann erhält man freilich ein blosses Spiel."
- (198)/

- (198) Ibid., p. 184. "... die Deutung der Formeln... besteht in gar nichts anderem als in der Eingliederung der Regeln der Kalküls in einen weiteren syntaktischen Zusammenhang."
- (199) Critical Notice of Introduction to Mathematical Thinking, Mind (1953), p. 544.
- (200) Weismann, op. cit., p. 183. "Aber wenn man mich fragt, was die Formel ' $1 + 1 = 2$ ' bedeutet, so werde ich nicht mit einer Schilderung meines Geisteszustandes antworten, sondern mit einer Zeichenerklärung."
- (201) Ibid., p. 185. "Weil ein Schachzug keine Gedanken ausdrückt, kann man von ihm keine Anwendung machen." Sollte es nicht richtiger heißen: weil wir keine Anwendung vorgesehen haben, drückt ein Schachzug keine Gedanken aus?"
- (202) Ibid., p. 184. "Mathematik ist es dann, wenn die Gleichung zum Übergang von einem Satz zum anderen verwendet wird; und sonst ist es Spiel." Cf. also p. 94, and Wittgenstein, Remarks, V-6.
- (203) Remarks..., op. cit., II-46, V-38, V-46, V-49, V-46.
- (204) Ibid., IV-2.
- (205) Ibid., II-30, II-29.
- (206) Ibid., II-36.
- (207) Ibid., I-82, I-84, Cf. also V-6.
- (208) Ibid., II-42, V-5, III-30, III-31, II-71, II-30.
- (209) Ibid., II-28.
- (210) Ibid., II-22.
- (211) Ibid., II-39.
- (212) Cf. ibid., II-50.
- (213) Cf. ibid., II-73.
- (214) Ibid., II-62.
- (215)/

- (215) Philosophical Investigations, op. cit., p. 224^e.
- (216) Ibid., p. 225^e.
- (217) Cf. Remarks..., op. cit., II-25.
- (218) Ibid., V-19.
- (219) Cf. ibid., IV-9 - IV-20.
- (220) Ibid., V-19.
- (221) Philosophical Investigations, op. cit., par. 197.
- (222) For an example of a proposition proved either with or without the aid of an Axiom of Choice, cf. Rosser, Logic for Mathematicians (New York: McGraw-Hill, 1953), pp. 515-516.
- (223) (a) Remarks..., op. cit., II-61.
 (b) Ibid., V-7.
 (c) Ibid., V-49.
 (d) Ibid., IV-7.
- (224) Cf. ibid., I-164.
- (225) Ibid., II-21. Cf. also II-43.
- (226) Ibid., III-41. Cf. also I-153, II-1ff.
- (227) Cf. ibid., I-157 - I-164, II-39, II-69.
- (228) Ibid., V-38.
- (229) Ibid., V-50.
- (230) Cf. ibid., II-69ff.
- (231) Ibid., III-8.
- (232) Calculating prodigies seem to have the use of a multiplication table up to 1000 x 1000. In Investigations (par. 236) occurs the remarks: "Calculating prodigies who get the right answer but cannot say how. Are we to say that they do not calculate? (A family of cases)." But, in fact, most calculating prodigies can say how (cf. e.g. Rouse Ball, Mathematical Recreations and Essays, 10th edn., ch. XIII.).
- (233)/

- (233) Remarks..., op. cit., II-66.
- (234) Philosophical Investigations, op. cit., par. 199. Cf. also
pars. 150, 557, 692; and Remarks, I-133, II-46.
- (235) Remarks..., op. cit., II-80.
- (236) O'Connor, 'Validity and Standards', op. cit., p. 218.
- (237) 'Why Are the Calculuses...', op. cit., pp. 21-22.
- (238) Ibid., pp. 22-23. Cf. also p. 24.
- (239) Ibid., p. 28.
- (240) Ibid., p. 26. Cf. e.g. sentence (c) in group (III) (p. 99
above).
- (241) Ibid., p. 21.
- (242) Cf. ibid., pp. 43-44.
- (243) Ibid., p. 31.
- (244) Introduction to Logic, 2d edn., transl. by Helmer (New York:
Oxford, 1946), p. 47 (my italics).
- (245) Review of Suppes, Introduction to Logic, Australasian Journal
of Philosophy (1958), p. 148.
- (246) 'The Rules of Natural Deduction', Analysis (1958), pp. 27-28.
- (247) Introduction to Mathematical Logic (Princeton, 1956), p. 172.
- (248) Mathematical Logic, rev. edn. (Harvard, 1951), pp. 171-172.
- (249) Church, op. cit., p. 72.
- (250) Cf. Prior, Formal Logic, op. cit., p. 304; Tarski, Logic,
Semantics, Metamathematics (Oxford, 1956), pp. 54-59.
- (251) There is much more to be said on this topic. My argument
(iv) could, I think, be developed at greater length, and
more technically.

(252)/

- (252) 'Why Are the Calculuses...', op. cit., pp. 32-33.
- (253) Cf. ibid., pp. 33-34.
- (254) "'If", "So", and "Because"', in Philosophical Analysis, ed. Black (Cornell, 1950), p. 334. Cf. also p. 332: "The still somewhat nebulous notion of application may now be clarified...".
- (255) Ibid., p. 328.
- (256) Ibid., p. 331.
- (257) Ibid., p. 333. Cf. also The Concept of Mind, op. cit., pp. 299-304.
- (258) "'If", "So", and "Because"', op. cit., p. 336.
- (259) Ibid., p. 332.
- (260) Ibid., p. 333.
- (261) Cf. Welsh and Clark, 'Applying Principles of Inference', Analysis (1956), pp. 14-20; Clark, 'Natural Inference', Mind (1956), pp. 455-472. Cf. also Brown, 'Misconceptions of Inference', Analysis (1955), pp. 135-144, and 'What the Tortoise Taught Us', Mind (1954), pp. 170-179. In 'Misconceptions of Inference' Brown says, inter alia, that the complexity of use of 'apply' and 'application' is "underestimated": this is an understatement.
- (262) Cf. 'On Conditionals', in Logical Studies (London: Routledge & Kegan Paul, 1957), p. 130.